COMPARISON OF EXPERIMENTAL AND THEORETICAL MODES OF OSCILLATION IN NON-NEUTRAL ELECTRON PLASMAS

by

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ABSTRACT

COMPARISON OF EXPERIMENTAL AND THEORETICAL MODES OF OSCILLATION IN NON-NEUTRAL ELECTRON PLASMAS

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We have measured the diocotron and Trivelpiece-Gould mode frequencies, radial density profile, and central temperature in a long (0.6 m), cylindrical Malmberg-Penning electron trap at four different magnetic field strengths. The total particle count varied by a factor of 10 and the magnetic field varied by a factor of 3.5. The temperatures were fairly constant. Using an equilibrium code (EQUILSOR), a 2-D particle-in-cell code (RATTLE), and a 3-D particle-in-cell code (INFERNO) we have calculated the frequencies corresponding to the experimental conditions. We found the codes gave results consistent with experiment in cases of high magnetic fields and high density profiles when the code's convergence criteria was satisfied. When low fields are analyzed, with resulting low densities, the codes did not agree with experiment.

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Chapter 1

Introduction

1.1 Motivation

In our attempt to understand the universe around us, we tend to make models of the things we are currently trying to understand. This is true in the case of electron non-neutral plasmas. Computer codes have been made that model certain oscillations inside an electron non-neutral plasma. The three computer codes that I used for my thesis are EQUILSOR, RATTLE, and INFERNO.

EQUILSOR was developed here at BYU by Professor Ross Spencer around 15 years ago. This code calculates the plasma density distribution when all that is known about the plasma are experimentally measurable quantities[1]. These quantities include the conductor geometries within the plasma confinement region, the potentials measured on the conductors, and the line integrated densities of the plasma.

Around the early 1990's, Professor Ross Spencer and K. C. Hansen, a BYU graduate student, developed RATTLE, a two-dimensional particle-in-cell code. This code was specifically written to watch the dynamics of the plasma at the particle level[2,3]. RATTLE follows all motions within the plasma except the cyclotron motion, it also assumes the plasma is axially symmetric.

INFERNO is another particle-in-cell code written less than five years ago. This code was written by Professors Ross Spencer and Grant Mason[4]. INFERNO was written as an extension to RATTLE but removes the assumption of axial symmetry made by RATTLE. By removing this assumption, INFERNO was written as a three-dimensional code.

These codes have proven useful in the understanding of non-neutral plasma physics. My advisor, Professor Bryan Peterson, along with Professor Grant Hart, feel these codes have been improperly used in some cases where they do not apply. The purpose of this thesis is to verify the region of validity of these codes. To do this, I experimentally measured the frequencies of the diocotron mode and Trivelpiece-Gould modes of oscillation at different plasma densities and compared these frequencies to those found using EQUILSOR, RATTLE, and INFERNO.

1.2 Theory behind the research

Trivelpiece-Gould modes of oscillation are electrostatic fluctuations that oscillate axially in cylindrical magnetized plasmas[5]. The diocotron mode is the circular movement of the center of the plasma around the axis of the trap when the plasma is off axis. The frequencies of the Trivelpiece-Gould modes and the diocotron mode are useful for diagnostic reasons. These modes are always in non-neutral plasmas. Although they are usually small, we intend to measure and use them to help us know other characteristics of the plasma. This is possible due to the relationship between the modes and the characteristics within the plasma. The computer codes that we have used in this study analyze the behavior of the electron plasma when given certain parameters. These codes allow us to determine the properties of plasmas, making plasmas easier to study.

The Trivelpiece-Gould modes of oscillation have harmonics that behave differently than sound waves. The higher mode harmonics in a plasma are not located at the expected integer multiple of a fundamental mode. Instead they occur slightly lower in frequency than the expected integer multiple would give.

By comparing the measured frequencies to those given from EQUILSOR, RATTLE, and INFERNO, I will verify the range of validity for the codes. In chapter two of this thesis I will expand on the experimental setup used to measure the frequencies of interest. Chapter three is devoted to the analysis involved with EQUILSOR, RATTLE, and INFERNO. The results of this research are reported in chapter four. I conclude in chapter five that the codes are valid for plasmas in high magnetic fields and with high densities. When a low magnetic field is used to confine a low density plasma, the codes are not valid.

Future experiments will be able to use the information found in this thesis to help diagnose why the codes did not work in the low magnetic field, low density case. From this diagnostic, the codes will potentially be improved upon and the range of validity will expand.

Chapter 2

EXPERIMENTAL SETUP

2.1 Malmberg-Penning trap

A Malmberg-Penning trap is a long cylindrical tube with a solenoid around it. A current is sent through the solenoid to produce an axial magnetic field inside the solenoid. The magnetic field is used to confine the plasma in the radial direction. Inside the solenoid is a section where the plasma resides. This section is made up of hollow rings that are used to study the plasma[See Figure 2.1]. A large negative electric voltage is applied to rings one and nine to confine the plasma axially. This section is shorter in length than the solenoid, so the solenoid can produce a nearly uniform magnetic field where the plasma is located. Our solenoid is 1.25 meters long. The inner rings have a total length of 0.6 meters. In this experiment we used magnetic field strengths of 0.02 Tesla, 0.03 Tesla, 0.05 Tesla, and 0.07 Tesla produced by the

Figure 2.1: Cylindrical Malmberg-Penning Trap: This is a diagram of (a) where the rings are located with dimensions and (b) where the plasma resides within the rings.

solenoid.

We have found that the behavior of the plasma depends on the quality of vacuum used in the experiment. Before we make any measurements on the plasma we need the background gas pressure around the plasma to be low. To accomplish this we allow the experimental setup to run for at least eight hours. This allows the magnet to stabilize and the background pressure to decrease sufficiently. We used an ion pump to pump the background pressure inside the trap to below 10^{-8} Torr.

Applying a voltage across a tungsten filament provides the electrons for the plasma. This filament is doped with thorium to increase the number of electrons the filament emits. In order to fill the trap with electrons, we start by grounding ring nine. Rings eight through two are also grounded. Ring one is kept at a large negative voltage, thus confining the electrons axially. The electrons are given enough energy from the voltage applied to the filament to escape from the filament and be attracted to the grounded rings. This process is known as applying a forward bias to the filament. The magnitude of this forward bias is determined by the relationship between the negative voltage on the filament to the grounded rings. By modifying the magnitude of the forward bias we can adjust the number of electron that make up our plasma and thus adjust the density of our plasma. After a certain time a large negative voltage in applied on ring nine and a positive voltage is applied to the filament. This configuration is called a reverse bias and keeps any additional electrons from entering the trap.

Since we are studying electron non-neutral plasmas, there is a net negative charge in our confinement region. This charge from the plasma produces an image charge on the surrounding rings. Although we keep the majority of these rings grounded so as to not disturb the plasma, we can study the fluctuations in charge on a ring to find out how the plasma behaves. This is how we are studying the electron non-neutral

Figure 2.2: Charge Collector Rings: These rings are numbered 1–10 starting with the center ring.

plasma.

After the plasma has been used in the study, the end confining potential on ring one is grounded to allow the electrons in the plasma to escape. Concentric charge collector rings are placed on the other side of ring one to collect the electrons as they exit the trap [See Figure 2.2]. The charge that these collector rings build up allows us to determine the radial density profile of the plasma as it exits the trap. This data assumes we have an azimuthal symmetry in our plasma and that our plasma is centered inside the trap. By factoring the length of the plasma into this information we can find the average density of the plasma at a given radius away from the center of the trap. This is called a line integrated density profile. The line integrated density profile made from this measurement is needed in EQUILSOR so that we can compare the experimental values with the values the computer codes give us.

2.2 Experimental Measurements Made

2.2.1 Diocotron Measurement, Density Profile, Temperature Measurement

The first measurement made is the frequency of the diocotron mode in the plasma. This mode relates to the position of the center of the plasma. If the center of the plasma is not at the center of the trap, the plasma starts to rotate around the center of the trap. This oscillation is known as the diocotron mode. The diocotron mode frequency is calculated through a relationship of the total number of particles in the

Figure 2.3: Typical Density Profile

plasma divided by the magnetic field strength. There is a shift in the central density of the plasma away from the center of the trap. This shift is in the r, θ plane of the plasma and goes as $\cos(m\theta)$. We have been studying the m=1 diocotron mode in our plasma.

The diocotron mode is known as a negative energy mode. This means that as we take energy away from the system, the mode will grow. This is a concern for our measurements, because we remove energy in our observation of the diocotron mode. To counter for this growth, energy is put back into the system in a way to prevent to diocotron mode from growing[6]. Adding energy back to the system to stop the diocotron mode from growing is known as diocotron feedback. This process is done so well at times that the diocotron mode will disappear. To observe the diocotron mode, while keeping it under control, we adjust the gain level on the diocotron feedback.

Once the diocotron mode has been measured, the trap releases the plasma and the end collector rings determine the radial density of that plasma as described above. A typical density profile can be seen in Figure 2.3.

The temperature of the plasma is determined using a slow change in the voltage on the dump gate. This slowly raises the large negative voltage on ring one to a value closer to ground. As the voltage becomes less negative, high energy electrons make it out of the plasma first and are collected by the collector rings at the end. A computer program was then used to compare the change of the voltage on the dump gate to the rate charge was collected due to the exiting electrons. Since the majority

Figure 2.4: Typical Trivelpiece-Gould Modes: Modes 1, 2, 4, and 5 are shown here. Mode 3 cannot be seen due to the placement of the detection ring.

of the density is found in the center, we collect the charge from the two central charge collector rings. The comparison of the voltage to the charge collected has been found to relate to the temperature of the plasma[7]. In this way we were able to determine the temperature of our plasmas.

2.2.2 Trivelpiece-Gould Modes of Oscillation

Trivelpiece-Gould modes of oscillation (T-G modes) are a particular class of electrostatic fluctuations within the electron non-neutral plasma. In this study we are only including the axisymmetric T-G modes and are also ignoring any axisymmetric T-G modes with radial nodes. T-G modes depend on the density, temperature, and the exact density profile of the plasma. We are able to detect the first, second, fourth, and sometimes the fifth T-G modes as we observe the plasma from ring three. (Refer back to Figure 2.1 for ring placement.) We are not able to see the third T-G mode because ring three is located at a node for that mode.

A spectrum analyzer was used to sweep through our frequency range from 1 MHz to 10 MHz. A typical sweep can be seen in Figure 2.4. Lightly damped T-G modes are normally excited enough by thermal oscillations within the system. If the T-G modes are heavily damped, we have to excite them with a function generator connected to ring 7 to see them. We detected the modes from the signal generated on ring three. The function generator is used to sweep the driving voltage through the same frequency range as the spectrum analyzer. If a T-G mode is present at the frequency of the driving voltage then the amplitude of the mode will increase as seen in the spectrum analyzer. This allows us to find the T-G modes and separate them from the electronic noise from the equipment and other signals from non-plasma sources. Typical driving voltages were on the scale of 1 mV – 10 mV. It should be noted that the potential of the plasma is on the order of 30 volts. This means we are driving the plasma with a voltage on the order of 100 to 1000 times smaller then the plasma potential. We drove the plasma with the smallest driving voltage possible while still being able to see the T-G modes. A higher drive will cause non-linear effects in the plasma making the T-G modes frequencies shift.

Chapter 3

ANALYSIS

3.1 Codes Used to Model the Plasma

3.1.1 EQUILSOR

Once the experimental measurements were made, we were able to compare them with the known computer codes. The first code we compared to was EQUILSOR. EQUILSOR needed the line integrated density profile and the temperature of the plasma as inputs. EQUILSOR takes these inputs and finds the electrostatic equilibrium condition of the plasma within the trap. It then produces a file giving this equilibrium condition. We call this file the equilibrium file. While making this file, EQUILSOR calculates the diocotron frequency. Thus, as part of its outputs, EQUILSOR produces the diocotron mode frequency and the equilibrium file.

EQUILSOR works by adjusting the charge density in the plasma until it is in electrostatic equilibrium. Thermodynamic equilibrium is also reached along the magnetic field lines within the plasma. It should be noted that this is not a global thermodynamic equilibrium. Since electrostatic equilibrium needs to exist everywhere within the confinement area, EQUILSOR models the trap using a grid. The density at each grid element is adjusted in EQUILSOR until an equilibrium is reached. The spacing between the grid points needs to be small enough to correctly represent all the rings of the trap and to resolve the Debye length, or the length away from a particle you can be and still distinguish the particle from its surroundings. Once this grid spacing is small enough, EQUILSOR can represent the ring placements properly. We observed that with smaller grid spacing, EQUILSOR could converge using a smaller accuracy limit. For our analysis we used 80 radial grid points with 750 axial grid points. We found an equilibrium to within the error of 10^{-3} of the given parameters. See Appendix A for a sample of the different input run files that were used for EQUILSOR [1].

3.1.2 RATTLE

The equilibrium file made by EQUILSOR is used in RATTLE. RATTLE is a two-dimensional particle-in-cell code. This means that it follows the paths particles make as they move around in a two-dimensional grid that represents the volume inside the trap. Since RATTLE moves the particles around the grid and follows their paths, we want to start the particles in equilibrium with their surroundings. This is why we use the equilibrium file from EQUILSOR as an input to RATTLE. This file guarantees the plasma will start in equilibrium. With each equilibrium file that we used, we had to also input the specific parameters for that file into RATTLE. To do this we made a different file with these correct parameters as inputs for RATTLE. See Appendix B for a sample of the different input files used to run RATTLE[2,3].

While looking for the T-G modes we found strong damping in the fourth and fifth T-G modes. To allow us to still see these T-G modes, we had to start the program in a manner that should excite the mode that we are looking for. RATTLE then uses a numerical filter to make detection sensitive to a particular mode value. The resulting files that RATTLE made could then be analyzed using MATLAB to find the modes of oscillation. Due to the mode-specific excitation and the numeric filter used to find the frequency, a different file was created for each different T-G mode.

3.1.3 INFERNO

INFERNO is a three-dimensional particle-in-cell code. It works similar to RATTLE in the analysis, but INFERNO uses a three-dimensional grid where RATTLE only uses in a two-dimensional grid because RATTLE assumes azimuthal symmetry[4]. We used INFERNO to help analyze data regarding the diocotron mode. We specifically tested the data where EQUILSOR failed to give an accurate value.

3.2 Understanding Codes with MATLAB

Once the files were generated using EQUILSOR and RATTLE they were analyzed with the help of MATLAB. MATLAB codes [Appendix D] were used to fit a Gaussian profile matching the following equation

$$f = \frac{f_o}{\sqrt{\pi\sigma}} \exp\left(-\frac{(\omega - w_o)^2}{2\sigma^2}\right) \tag{3.1}$$

to the measured diocotron mode frequency sweep. [See Figure 3.1]

We fit a Gaussian profile to the diocotron mode because the diocotron mode is not damped. A Lorentzian profile of the following equation

$$f = \frac{f_o}{\gamma} \frac{\gamma/2}{\sqrt{(\omega - w_o)^2 + (\gamma/2)^2}} \tag{3.2}$$

was used to fit each T-G mode sweep in the experimentally measured modes of oscillation. [See Figure 3.2] T-G modes are damped, so the Lorentzian profile fit these modes better. These codes allowed us to find the most accurate value for our experimental data.

With each file created from RATTLE for the different T-G modes of oscillation we found the frequency that best fit the oscillation behavior of the mode. [See Figure 3.3] This was difficult at times due to the heavy damping that was observed in the higher T-G modes and in all the T-G modes at the lower magnetic field cases. The

Figure 3.1: MATLAB fit of Diocotron Mode: The smooth line is the MATLAB fit of the data collected (asterisks).

Figure 3.2: MATLAB fit of Trivelpiece -Gould Modes: The smooth line is the MATLAB fit and the raw data is marked by asterisks.

heavy damping in some cases caused the modes to only complete a few oscillations before being damped away.

Figure 3.3: MATLAB fit of RATTLE Data: Two oscillation modes are seen here with a fit signal for each; (a) Mode 1, (b) Mode 2. There is visible damping in Mode 2 as seen in (b).

Chapter 4

RESULTS

As referred to above, we used four different magnetic field strengths: 0.02 T, 0.03 T, 0.05 T, and 0.07 T. Depending on the strength of the magnetic field, we would get different density profiles. At lower magnetic field strengths we were getting lower density profiles. This may be a result of losing more electrons to the confinement rings during the setup of the plasma. A characteristic density profile of each magnetic field strength is displayed in Figure 4.1 (a)–(d). Notice that the central density of the 0.07 T density profile is about 3.5 times larger then the central density of the 0.02 T density profile. This feature is significant to point out because we input the profile data into EQUILSOR. This means that with a different profile we will get different results from EQUILSOR.

Figure 4.1: Density Profiles: (a) 0.07 T, (b) 0.05 T, (c) 0.03 T, (d) 0.02 T, (e) 0.07 T Low Density, (f) 0.07 T Lowest Density

4.1 Diocotron Modes

We found that EQUILSOR consistently produced values for the diocotron mode that were lower than the experimentally measured values. These values were less than 10% different from the measured values for the higher three magnetic field cases. We note that the diocotron mode inversely depend on the magnetic field, or 1/B. In this experiment we have at least a 10% uncertainty in the magnitude of the magnetic field. The results of EQUILSOR fit within the error of our measurements. We conclude that these comparisons are consistent with experimental values.

For the 0.02 T case we found around 30% discrepancy. This discrepancy made us look closer at the conditions on the plasma. As we looked closer we realized a condition that needed to be satisfied was not satisfied by the 0.02 T case. This condition is as follows,

$$\frac{\lambda_D^2}{2r_pZ_p} < 0.01 \tag{4.1}$$

where λ_D is the Debye length, r_p is the plasma radius, and Z_p is half the plasma length. Since this condition was not met by the 0.02 T case we do not expect EQUILSOR to give good results. We note one more time that it did not give good results.

In order to determine if the magnetic field was causing the discrepancy in the results, we took more data with the magnetic field at 0.07 T but adjusted the forward bias voltage used to fill the trap with electrons. We adjusted the voltage so that the density of the plasma is similar to those for the 0.02 T and 0.03 T cases. A characteristic density profile can be seen in Figure 4.1 e and f. We used EQUILSOR as described above for these data sets.

For the density profile similar to the 0.03 T case, called "Low Density," we found that EQUILSOR gave values of the diocotron frequency within 10% of the experimental values. This is similar to the 0.03 T case.

For the density profile similar to the 0.02 T case, called "Lowest Density," we found

Figure 4.2: N_{tot}/B vs. Diocotron Frequency. The circled values in the lower left are the experimental and theoretical values from the 0.02 T case.

that EQUILSOR gave frequencies with a 20% discrepancy. Although these values are closer together then the 0.02 T case, they still do not match within experimental uncertainties. The profile for the "Lowest Density" is different from that of the 0.02 T case but not to the point where EQUILSOR will produce values consistent with experimental results.

A comparison graph of the measured values of the diocotron mode is found in Figure 4.2. Take note of the linear fit to the experimental values and to the values produced from EQUILSOR. There should be a linear relationship between the frequency of the diocotron mode to the total particles divided by the strength of the magnetic field. The values in the lower left that are circled are the values from the 0.02 T magnetic field strength. Notice that these affect the linear fit of the rest of the values.

As mentioned above, INFERNO was used to analyze the data where the condition of validity for EQUILSOR was not met. This means we used INFERNO to analyze the 0.02 T magnetic field data. INFERNO was able to follow the particles that made up the plasma in a three dimensional grid. With these extra capabilities found within INFERNO, we were able to calculate the frequency of the diocotron mode. We found INFERNO gave results that differed by 25% of the measured values. This is an improvement from the 30% discrepancy from EQUILSOR. Although these results are closer then the values produced by EQUILSOR, they are still not consistent with

Figure 4.3: Outside Resonance for (a) 0.03 T and (b) 0.05 T Cases. The arrows are pointing to the region where the outside resonance is strongest.

experimental values.

4.2 Trivelpiece-Gould Modes

As described in section 3.1, we used the equilibrium file produced by EQUILSOR as part of our input into RATTLE. We used RATTLE to find a value for each T-G mode that we could experimentally find. The third T-G mode was not seen experimentally due to the placement of the detector ring. The detector ring that we used to watch the T-G modes in the plasma is located at a node for the third T-G mode.

In all the cases, except the 0.07 T case, we noticed an odd peak in the spectrum located around the fourth and fifth T-G modes. This peak is a resonant effect caused from the electronics located outside the plasma itself, which we will call an outside resonance. See Figure 4.3 for an example of this outside resonance for the 0.03 T and 0.05 T cases. This outside resonance is due to a resonance between our detector ring and the amplifier used to detect the signal from the ring. In the 0.05 T case we were still able to measure a fifth T-G mode due to the magnitude of its peak. For the 0.02 T, 0.03 T, "Low Density," and "Lowest Density" cases we were not able to find the fourth and fifth T-G modes in the noise of the outside resonance.

In the 0.02 T, 0.03 T, Low Density, and Lowest Density cases we also noticed non-linear effects from the T-G modes we measured. These effects shifted the frequencies lower. The drive voltage that we used causes this non-linear effect. The higher the drive voltage, the larger the shift would be in the T-G modes. It should be noted that

Figure 4.4: T-G Mode Fitting: (a) 0.07 T, (b) 0.05 T, (c) 0.03 T, (d) 0.02 T, (e) 0.07 T Low Density, (f) 0.07 T Lowest Density

without any drive voltage we would not be able to see any T-G modes for these cases. We made our comparisons with the values from the smallest drive voltage used. In analyzing these data points in RATTLE, we found heavy damping in the T-G modes as well. This made finding accurate frequencies of the T-G modes difficult.

We found the comparisons of the T-G modes in the 0.07 T, 0.05 T, and 0.03 T cases are consistent with each other with a small discrepancy around 1–3%, as seen in Figure 4.4. It should be noted that for the 0.02 T magnetic field case the discrepancy ranged as high as 17–20%. Small discrepancies were also found in the Low Density and Lowest Density cases ranging as high as 5–6%. We conclude that RATTLE produces frequencies of the T-G modes that are consistent with experimentally measured values, except in low magnetic field cases with low density.

As described above, we had RATTLE look for one T-G mode at a time. While looking for the mode we wanted, RATTLE would produce beating patterns in the frequencies around the desired frequency. This beating is due to the T-G modes converting their energy into one another[8]. Thus they are constantly changing their amplitude in the plasma. RATTLE saw this changing amplitude as it looked for the modes and treated it as several frequencies beating around each other. The values of the beating frequencies can be averaged to find the value of the mode frequency. Thus RATTLE will still be able to give results for the desired T-G mode. The perceived beating that is found by RATTLE, but not found in the experiment, should be noted for anyone that will use RATTLE.

Chapter 5

CONCLUSION

We conclude that EQUILSOR, INFERNO, and RATTLE model an electron non-neutral plasma well. Equation 4.1, involving the Debye length, the plasma radius, and the plasma length, must be satisfied in order for EQUILSOR to produce accurate frequencies of the m=1 diocotron mode. As long as this equation was satisfied, an appropriate grid spacing within the program can be found to model the plasma correctly. When the equation was not satisfied, INFERNO was able to improve the discrepancy between experimental and theoretical values from the 30% in EQUILSOR to 25%.

RATTLE was used to compare the frequencies of the T-G modes to theoretical values. By having RATTLE look for T-G modes one at a time, we were able to distinguish the frequencies of the modes from other noise and non-plasma signals detected by the equipment. Damping was seen and increased as we went to lower magnetic fields and lower densities of the plasma. This damping made finding the frequencies of the T-G modes mode difficult. A resonance between the detection ring and the amplifier used to detect the signal appeared around the frequency of the fourth T-G mode. This resonance made analyzing the fourth and sometimes the fifth

T-G modes impossible in most cases.

Our high density profiles with high magnetic fields produced robust results when comparing the frequencies measured and calculated. In low density profiles with low magnetic fields, for us the 0.02 T case, great care had to be taken to ensure validity of the codes. The discrepancies found in the 0.02 T cases could be from misunderstanding of the measurements made in the plasma. Caution needs to be taken to correctly model the plasma when accounting for the numerical grid structure in the convergence criteria of the codes. Overall, the codes, EQUILSOR, RATTLE, and INFERNO, give results that correlate well with experimentally measured values when we were careful to make the underlying assumptions of the codes valid. If the appropriate care is not taken, the codes will still produce values but will not accurately represent the plasma.

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Appendix A

EQUILSOR Codes

Copy of run files used in EQUILSOR

With each run file I used a unique name known as a character run ID.

The four magnetic field strengths that I used are 0.02, 0.03, 0.05, and 0.07. Tempstart and tempend varied with each run according to the temperature of that set of runs.

```
30Jn04.000          Enter the 10--character run ID
-1.6e-19, 9.11e-31 Enter q and m for the particles
0.07                Enter the magnetic field in Tesla
plmeta              Enter the plot type: xwin, plmeta, ps, or none
ringdata            Enter the equilibrium type: thermal, midplane, ringdata,
full                Enter the equilibrium size: half or full
80, 750             Enter nr, nz
0,0.05, -0.374, 0.374 Enter rmin, rmax, zmin, zmax
5000, 1.0e-3, 1.75 Enter itmax, etest, and alpha (underrelaxation factor)
1                   Enter initype
1e12, 3.07, 3.07, 100 Enter peak density, tempstart, tempend, ntemp
20, 1, 0.5          Enter ncon and icompres and edgefac (for shape cutoff)
0.3, 0.2, 0.0       Enter zlen, rmid, and zcenter (guess for ellipse)
```

```
0, 0, 0             Enter Vleft, Vright, Vwall
-0.31, 0.31, 0.0375 Enter zcutlef, zcutrgt, and rcut
2                   Enter the number of conducting rings at the wall
-0.33, -0.305, 0    Enter the ring z-values and voltages:
0.305, 0.33, 0
7                   Enter the number of internal conductor segments
0.05, -0.3451, 0.04, -0.3451, 0 Enter the segment endpoints and voltages:
0.04, -0.337, 0.04, -0.3451, 0
0.04, -0.333, 0.04, -0.305, -140
0.04, -0.301, 0.04, 0.301, 0
0.04, 0.305, 0.04, 0.333, -140
0.04, 0.337, 0.04, 0.3451, 0
0.04, 0.3451, 0.05, 0.3451, 0
```

Appendix B

RATTLE Codes

Copy of run files used in RATTLE

B.1 Mode 1

```
runfile.run

0 ; read equilibrium file (0) or restart file (1)

0 ; read flint file(0=no,1=yes,2=Dubin-seed);next are input,output names

30Jun04.000.fil

30Jun04.000.1

plmeta          ; plot type (plmeta,xwin, tekf)

2 17            ; r,z grid point to view density array

1               ; plot to 0=screen, 1=file

16000 .50e-9 1000 5     ; nsteps, dt, numplot, numdiag

0. 0.           ; omega, delta

0.0000          ; zstretch (fractional)

1 2e-4          ; mz, zampl for mode seeding

.04             ; rwall

-3.2e6 3.2e6    ; vmin, vmax

1000000         ; number of particles
```

```
3.7 7 0e-4      ; scalev, scalez, zoffset (meters)
```

B.2 Mode 2 For high density profiles

```
runfile.run

0 ; read equilibrium file (0) or restart file (1)

0 ; read flint file(0=no,1=yes,2=Dubin-seed);next are input,output names

30Jun04.000.fil

30Jun04.000.2

plmeta          ; plot type (plmeta, xwin, tekf)

2 17            ; r,z grid point to view density array

1               ; plot to 0=screen, 1=file

4000 .50e-9 1000 5  ; nsteps, dt, numplot, numdiag

0. 0.           ; omega, delta

0.0000          ; zstretch (fractional)

2 4e-4          ; mz, zampl for mode seeding

.04             ; rwall

-3.2e6 3.2e6    ; vmin, vmax

1000000         ; number of particles

3.7 7 0e-4      ; scalev, scalez, zoffset (meters)
```

B.3 Mode 2 For low density profiles

```
runfile.run

0 ; read equilibrium file (0) or restart file (1)

0 ; read flint file(0=no,1=yes,2=Dubin-seed);next are input,output names

09Jul04.003.fil

09Jul04.003.2

plmeta          ; plot type (plmeta, xwin, tekf)

2 17            ; r,z grid point to view density array

1               ; plot to 0=screen, 1=file
```

```
4000 .50e-9 1000 5  ; nsteps, dt, numplot, numdiag
0. 0.           ; omega, delta
0.0000          ; zstretch (fractional)
2 16e-4         ; mz, zampl for mode seeding
.04             ; rwall
-3.2e6 3.2e6    ; vmin, vmax
1000000         ; number of particles
3.7 7 0e-4      ; scalev, scalez, zoffset (meters)
```

B.4 Mode 4

```
runfile.run
0 ; read equilibrium file (0) or restart file (1)
0 ; read flint file(0=no,1=yes,2=Dubin-seed);next are input,output names
30Jun04.000.fil
30Jun04.000.4
plmeta          ; plot type (plmeta, xwin, tekf)
2 17            ; r,z grid point to view density array
1               ; plot to 0=screen, 1=file
4000 .50e-9 1000 5  ; nsteps, dt, numplot, numdiag
0. 0.           ; omega, delta
0.0000          ; zstretch (fractional)
4 16e-4         ; mz, zampl for mode seeding
.04             ; rwall
-3.2e6 3.2e6    ; vmin, vmax
1000000         ; number of particles
3.7 7 0e-4      ; scalev, scalez, zoffset (meters)
```

B.5 Mode 5

```
runfile.run
```

```
0 ; read equilibrium file (0) or restart file (1)
0 ; read flint file(0=no,1=yes,2=Dubin-seed);next are input,output names
30Jun04.000.fil
30Jun04.000.5
plmeta          ; plot type (plmeta, xwin, tekf)
2 17            ; r,z grid point to view density array
1               ; plot to 0=screen, 1=file
4000 .50e-9 1000 5  ; nsteps, dt, numplot, numdiag
0. 0.           ; omega, delta
0.0000          ; zstretch (fractional)
5 16e-4         ; mz, zampl for mode seeding
.04             ; rwall
-3.2e6 3.2e6    ; vmin, vmax
1000000         ; number of particles
3.7 7 0e-4      ; scalev, scalez, zoffset (meters)
```

Appendix C

MATLAB Codes

Copy of the MATLAB codes

C.1 dioc_freq_fit.m

```
%Fitting a gaussian profile to the diocotron mode of oscillation
clear; close all; clc;

asking = input('what file do you want to fit the diocotron to?');
infile = sprintf('c:/data/ModeComparison/%s',asking);

%reading in the file and assigning values for the information that I need
[format,names,sizes,fid,dependence]=read_header(infile);
plot_title=sprintf(asking); %title for plots

%Get the information to make a plot of the profile
rad = read_trace(fid,'radius',format,names,sizes); %radius
dens = read_trace(fid,'density',format,names,sizes); %density

plot(rad,dens,'k');
```

```
title(plot_title)
xlabel('radius (cm)')
ylabel('density (cm^ -^ 3)')

pause

%Reading in the amplitude for this run.
amp = read_trace(fid,'amplitude',format,names,sizes);

%Reading in the frequency for this run.
wraw = read_trace(fid,'frequency',format,names,sizes);

%set up for the plot of the fitting function at the end
wmin = min(wraw);
wmax = max(wraw);
npts = 1001;
dw = (wmax - wmin) / (npts - 1);
wplot = wmin:dw:wmax;

figure
plot(wraw,amp,'b-');
xlabel('frequency')
ylabel('amplitude (arbitrary)')

%fit to the form f = fo/sqrt(pi*sigma)*exp(-(w-wo)^ 2/(2*sigma^ 2))
%Parameters for fo
par(1) = 3e5; %input('What is your guess for fo?');
```

```
par(2) = input('What is your guess for wo?'); %Parameters for wo
par(3) = 1000; %Parameters for sigma
%Parameter for offset
par(4) = -20; %input('What do you think the offset is?');

%how was my guess
ftest = diocofuncfit(par, wraw);
plot(wraw,ftest,'r-',wraw,amp,'b-'); pause
option = optimset ('TolX',1e-7,'MaxFunEvals',100000);
par = fminsearch (@diocoleastsq, par, option, wraw, amp);
fplot = diocofuncfit(par, wplot);

%Plotting the end result with the initial data values
plot(wraw,amp,'b-',wplot,fplot,'r-');
xlabel('frequency')
ylabel('amplitude (arbitrary)')
title(plot_title)
%end dioc_freq_fit.m
```

C.2 diocoleastsq.m

```
%function diocoleastsq.m
function s = diocoleastsq (par, omega, f)
s = sum((f - diocofuncfit(par,omega)).^ 2);
%end diocoleastsq.m
```

C.3 diocofuncfit.m

```
%function diocofuncfit.m
function f = diocofuncfit (par, omega)
%Unpacking the parameters that are in par
```

```
fod = par(1);
wod = par(2);
sigmad = par(3);
offset = par(4);
fd = fod/sqrt(pi*sigmad)*exp(-1*(omega-wod).^ 2/(2*sigmad^ 2));
f = offset + fd;
%end diocofuncfit.m
```

C.4 leastsq.m

```
%function leastsq.m
function s = leastsq (par, omega, f)
s = sum((f - funcfit(par,omega)).^ 2);
%end leastsq.m
```

C.5 freqrun_fit.m

```
%Fitting a lorentzian per T-G mode for a frequency run
clear; close all; clc;

%readining in the file and assigning values from the
%information that I need
[format,names,sizes,fid,dependence]=...
    read_header('c:/data/ModeComparison/16Jul04.003');
plot_title=sprintf('16Jul04.003');

%Reading in the amplitude for this run.
amp = read_trace(fid,'ampl',format,names,sizes);

%Amplitude for a certain time step.
fraw = amp(1:80,350);
```

```
fraw = fraw';

%Reading in the frequency for this run.
wraw = read_trace(fid,'freq',format,names,sizes);
wraw = wraw(1:80);

%set up for the plot of the fitting function at the end
wmin = min(wraw);
wmax = max(wraw);
npts = 1001;
dw = (wmax - wmin) / (npts - 1);
wplot = wmin:dw:wmax;
plot(wraw,fraw,'b-');
pause

%We want to fit a function of the form
%f = fo./gamma.*(gamma/2)./(sqrt((omega-wo).^ 2+gamma.^ 2/4))
%for every mode of oscillation that we see.

%Parameters for fo: related to the amplitude
par(1) = 400; par(2) = 400; par(3) = 127; par(4) = 546;
%Parameters for wo
par(5) = 1.4e6; par(6) = 2.7e6; par(7) = 2.8e6; par(8) = 7.4e6;
%Parameters for gamma
par(9) = 400000; par(10) = 1e6; par(11) = 4.0939e5; par(12) = 1.1668e5;
%Parameter for offset
par(13) = -0.000169;
```

```
%We see a fifth mode in some runs so here are the parameters for it
par(14) = 2471; %fo
par(15) = 8.8e6; %wo
par(16) = 1.1668e5; %gamma

%how was my guess
ftest = funcfit(par, wraw);
plot(wraw,ftest,'r-',wraw,fraw,'b-');
pause

option = optimset ('TolX',1e-7,'MaxFunEvals',100000);
par = fminsearch (@leastsq, par, option, wraw, fraw);
fplot = funcfit(par, wplot);

%Plotting the end result with the initial data values
plot(wraw,fraw,'b-',wplot,fplot,'r-');
xlabel('frequency')
ylabel('amplitude')
title(plot_title)
%end freqrun_fit.m
```

C.6 funcfit.m

```
%function funcfit.m
function f = funcfit (par, omega)

%Unpacking the parameters that are in par
fo1 = par(1);
fo2 = par(2);
```

```
fo3 = par(3);
fo4 = par(4);
fo5 = par(14);

wo1 = par(5);
wo2 = par(6);
wo3 = par(7);
wo4 = par(8);
wo5 = par(15);

gamma1 = par(9);
gamma2 = par(10);
gamma3 = par(11);
gamma4 = par(12);
gamma5 = par(16);

offset = par(13);
f1 = fo1./gamma1.*(gamma1/2)./(sqrt((omega-wo1).^2+gamma1.^2/4));
f2 = fo2./gamma2.*(gamma2/2)./(sqrt((omega-wo2).^2+gamma2.^2/4));
%f3 = fo3./gamma3.*(gamma3/2)./(sqrt((omega-wo3).^2+gamma3.^2/4));
f4 = fo4./gamma4.*(gamma4/2)./(sqrt((omega-wo4).^2+gamma4.^2/4));
f5 = fo5./gamma5.*(gamma5/2)./(sqrt((omega-wo5).^2+gamma5.^2/4));
f = offset + f1 + f2 + f4 + f5;
%end funcfit
```

C.7 convert_to_gdata.m

```
%Converting ring density data to gdata.
x = 1;
```

```
while x == 1
clear; close all; clc;

asking = input('what file do you want to convert?');
infile = sprintf('c:/data/ModeComparison/%s',asking);

%readining in the file and assigning values for the information that I need
[format,names,sizes,fid,dependence]=read_header(infile);

%Get the information
rad = read_trace(fid,'radius',format,names,sizes); %radius
dens = read_trace(fid,'density',format,names,sizes); %density

%Converting the data to gdata
grad = rad * 0.01; %converting cm to m in ring size

%converting 1/cm^3 to 1/m^2 by multiplying by length 0.6 m
gdens = dens * 0.6 * 1000000 * 0.85;

outfile = sprintf('c:/data/ModeComparison/gdata%s',asking);
fid = fopen(outfile,'w');
fprintf(fid,'quadratic\r\n14\r\n');
fprintf(fid,'0\t %15.2f\r\n', gdens(1));

for n = 1:10
 fprintf(fid,'%1.5f\t %15.2f\r\n', grad(n),gdens(n));
end
```

```
fprintf(fid, '0.04\t 0\r\n0.045\t 0\r\n0.05\t 0\r\n');
fclose(fid);

x = input('Do you want to do this again? (1 = yes, 0 = no) ');
end
%end convert_to_gdata.m
```