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THE EFFECT OF Be^7 K-CAPTURE ON THE SOLAR NEUTRINO FLUX*

ICKO IBEN, JR., KENNETH KALATA, AND JUDAH SCHWARTZ
Massachusetts Institute of Technology
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ABSTRACT

All other things being equal, bound-electron capture by the Be^7 nucleus near the solar center decreases the calculated rate of B^8 neutrino emission from the Sun.

I. INTRODUCTION

Under conditions that prevail near the center of the Sun, Be^7 disappears several hundred times more rapidly by the $Be^7(e^-,\nu)Li^7$ reaction than by the $Be^7(p,\gamma)B^8$ reaction. The equilibrium abundance of Be^7 is thus essentially that obtained by equating the $Be^7(e^-,\nu)Li^7$ rate with the $He^3(He^4,\gamma)Be^7$ rate. The $Be^7(p,\gamma)B^8$ rate and, consequently, the $B^8(\beta^+\nu)Be^{8*}$ rate are then proportional to this equilibrium abundance.

It has been universally assumed that, under solar conditions, the Be^7 nucleus captures electrons solely from the continuum. There is, however, a finite probability that Be^7 exists as an atom with one or two bound K-shell electrons. The nuclear electron-capture probability is larger when bound electrons are taken into account and the calculated equilibrium abundance of the Be^7 nucleus is correspondingly reduced. The $Be^7(p,\gamma)B^8$ rate and, hence, the $B^8(\beta^+\nu)Be^{8*}$ rate are reduced by the same factor. Since the current experiment to detect solar neutrinos (Davis 1964; Bahcall 1966) is primarily sensitive to neutrinos from the $B^8(\beta^+\nu)Be^{8*}$ reaction, it is of interest to examine quantitatively the influence of K-capture by Be^7.

II. FIRST APPROXIMATION—NO SCREENING

The free-electron capture probability may be written as

$$\omega_f = \frac{|\psi_f(0)|^2}{2|\psi_{\rm lab}(0)|^2}\,\omega_{\rm lab}\,, \tag{1}$$

where $|\psi_f(0)|^2$ is the free-electron density at the Be^7 nucleus in the star, $2|\psi_{\rm lab}(0)|^2$ is the electron density at the Be^7 nucleus in a *neutral*, unscreened atom in the ground state, and $\omega_{\rm lab} = \ln(2)/53.6$ days $= 1.5 \times 10^{-7}$ sec^{-1}. We have, approximately,

$$|\psi_f(0)|^2 = n_e\,\langle 2\pi\eta\rangle\,, \tag{2}$$

where $n_e = \rho/(\mu_e M_{\rm H})$ = mean electron number density at a given point in the star, $\langle\eta\rangle$ = average over the electron Maxwell-Boltzmann distribution of $4e^2/\hbar v_e$. In these expressions, ρ = matter density (gm/cm^3), μ_e = electron molecular weight, $M_{\rm H}$ = mass of hydrogen nucleus (gm), e = electron charge (e.s.u.), $2\pi\hbar$ = Planck's constant (ergs/sec), and v_e = electron velocity (cm sec^{-1}).

Inserting the estimate for $|\psi_{\rm lab}(0)|^2$ given by Bahcall (1962), we have

$$\omega_f \cong 4.24 \times 10^{-9}(\rho/\mu_e)T_6^{-1/2}{\rm sec}^{-1}\,, \tag{3}$$

where T_6 = temperature in 10^6 ° K.

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We now examine, on the assumption that screening may be neglected, the extent to which Be^7 may be only partially ionized. For simplicity and without much loss in accuracy, we neglect all excited states. That is, we assume that only the ground state enters into the partition function for each ionization state. The probabilities f_1 and f_2 that one or two K-shell electrons are associated with any given Be^7 nucleus are

$$f_1 = \lambda/[1 + \lambda + 0.25\lambda^2 \exp(-\Delta\chi/kT)] ,$$
$$f_2 = 0.25\lambda [\exp(-\Delta\chi/kT)] f_1 , \tag{4}$$

where

$$\lambda = n_e(h^2/2\pi mkT)^{3/2} \exp(\chi_1/kT) . \tag{5}$$

Here k = Boltzmann's constant, m = electron mass, χ_1 = fourth ionization potential of the Be^7 atom = 216.6 eV, χ_2 = third ionization potential of the Be^7 atom = 153.1 eV, and $\Delta\chi = \chi_1 - \chi_2 = 63.5$ eV.

The probability per second that a Be^7 nucleus will capture an electron is now

$$\omega_0 = \omega_f[1 + f_1|\psi_1(0)/\psi_f(0)|^2 + f_2|\sqrt{(2)}\psi_2(0)/\psi_f(0)|^2] , \tag{6}$$

where $|\psi_1(0)|^2 = (\pi a^3)^{-1}$ = electron density at the nucleus for the triply ionized Be^7 atom in the ground state. Here $a = (a_0/4) = 0.25 \times$ (first Bohr radius for hydrogen). Since

$$\lambda|\psi_1(0)/\psi_f(0)|^2 = [(8e^2/2a)/kT] \exp(\chi_1/kT) = (5.07/T_6) \exp(2.515/T_6) , \tag{7}$$

we have

$$\omega_0 = \omega_f[1 + (5.07/T_6) \exp(2.515/T_6)S] , \tag{8}$$

where

$$S = [1 + 0.25\lambda \exp(-0.735/T_6)|\sqrt{(2)}\psi_2(0)/\psi_1(0)|^2]$$
$$\div [1 + \lambda + 0.25\lambda^2 \exp(-0.735/T_6)] , \tag{9}$$
$$\lambda = 0.246(\rho/\mu_e T_6^{3/2}) \exp(2.51/T_6) .$$

Assuming that $|\psi_2(0)| \sim |\psi_{\rm lab}(0)|$, so that

$$2|\psi_2(0)/\psi_1(0)|^2 \sim 1.74 ,$$

we have

$$S \cong [1 + 0.435\lambda \exp(-0.735/T_6)]/[1 + \lambda + 0.25\lambda^2 \exp(-0.735/T_6)] . \tag{10}$$

For small λ,

$$\omega_0/\omega_f \cong 1 + (5.07/T_6) \exp(2.515/T_6) , \tag{11}$$

whereas for large λ (two *or* more bound electrons),

$$\omega_0/\omega_f \cong 1 + 35.8\mu_e T_6^{1/2}/\rho . \tag{12}$$

Values of ω_0/ω_f at various points in a solar model constructed by Sears (1964) are shown in Table 1. Note that within the inner one-tenth of the Sun's mass, where most of the B^8 neutrino flux is produced, the average value of ω_0/ω_f is $\sim 4/3$. This means that the B^8 neutrino flux calculated by Sears should be reduced by this same factor (provided all other reaction rates are unchanged).

III. SECOND APPROXIMATION—WITH SCREENING

In the Debeye-Hückel approximation, the average potential presented to an electron by a nucleus of charge Ze is given by

$$V = -(Ze^2/r) \exp(-r/R) . \tag{13}$$

The screening radius R obeys

$$R^{-2} = (4\pi e^2/kT) \sum_j Z_j^2 n_{0j}\,, \tag{14}$$

where n_{0j} is the average number density of particles of charge $Z_j e$. In a medium composed primarily of hydrogen (abundance by mass X) and helium,

$$R \cong 0.63 \times 10^{-8} \left[\frac{4T_6}{\rho(3+X)}\right]^{1/2} = 1.19 \left[\frac{64T_6}{\rho(3+X)}\right]^{1/2} a\,. \tag{15}$$

In order to compute the effect of bound-electron capture, we must first obtain, as a function of the screening radius R, the ground-state energy and the wave function describing an electron in the field of the screened Be^7 nucleus. Before presenting results of exact solutions of the equation

$$H\psi = [-(\hbar^2/2m)\nabla^2 - (4e^2/r) \exp(-r/R)]\psi = E\psi\,, \tag{16}$$

it is worthwhile to illustrate the approximate nature of the solutions obtained by applying the variational principle.

TABLE 1

EFFECT OF BOUND ELECTRONS ON THE $Be^7(e^-,\nu)Li^7$ RATE

Mass Fraction	ρ	T_6	X	λ	ω_0/ω_f	R/a	σ_R	C^2_R	λ_R	ω_R/ω_f
0.0	158	15.7	0.36	0.498	1.294	1.64	0.203	0.642	0.438	1.172
0.1	83	12.8	.58	.428	1.385	1.98	.294	.735	.372	1.256
0.2	59	11.3	.65	.394	1.454	2.18	.334	.773	.340	1.312
0.3	43	10.1	.68	.355	1.531	2.41	.380	.805	.304	1.378
0.4	31	9.0	.69	.316	1.626	2.68	.426	.835	.270	1.458
0.5	22	8.1	.70	.273	1.741	3.00	.473	.860	.232	1.556
0.6	15	7.1	.71	.238	1.790	3.40	.521	.887	.201	1.605
0.7	9.4	6.2	.71	.192	2.097	4.02	.582	.912	.162	1.858
0.8	5.0	5.1	.71	.150	2.485	5.00	.651	.936	.126	2.190
0.9	1.8	3.9	0.71	0.094	3.340	7.28	0.745	0.973	0.080	2.950

As a trial function, we choose

$$\psi_a(r) = (\pi a^3)^{-1/2} \exp(-r/a)\,. \tag{17}$$

Minimizing the expression

$$E(a) = \langle \psi_a | H | \psi_a \rangle \tag{18}$$

with respect to variations in a, we find that the minimum value of $E(a) = E_R$ (= approximate ground-state energy) occurs when $a = a_R$, where

$$(a_R/a)[1 + (3a_R/2R)] = (1 + a_R/2R)^3\,. \tag{19}$$

Then

$$E_R = -(2e^2/a_R)(1 - a_R/2R)/(1 + a_R/2R)^3\,, \tag{20}$$

and the square of the trial wave function at the origin becomes

$$|\psi_R(0)|^2 = (a/a_R)^3 |\psi_1(0)|^2\,. \tag{21}$$

In Figure 1 the functions $\sigma_R = -E_R/\chi_1$ and $C_R^2 = (a/a_R)^3$ are plotted against the parameter (R/a). It is interesting that, even for relatively large screening radii, the effect of screening on the ground-state energy is considerable. Note that the bound-state energy vanishes when the screening radius equals the first Bohr radius.

Results of exact solutions to equation (16) are also shown in Figure 1. Note that, as might be expected, energies obtained by the variational technique are quite accurate over a wide range in (R/a), whereas the wave function at the origin is not approximated particularly well by the variational solution except for large screening radii.

We shall not attempt to derive energies and wave functions for the ground state of the doubly ionized Be^7 atom. Instead, we content ourselves with the approximations: $\chi_2 \to \sigma_R \chi_2$, $\Delta\chi \to \sigma_R \Delta\chi$, and $|\psi_2(0)|^2 \to C_R^2 |\psi_2(0)|^2$. The total K-capture probability for the Be^7 nucleus is then approximated by

$$\omega_R = \omega_f [1 + (5.07/T_6) \exp (2.515\sigma_R/T_6) S_R] , \tag{22}$$

where

$$S_R = C_R^2 [1 + 0.435\lambda_R \exp (-0.735\sigma_R/T_6)]/[1 + \lambda_R + 0.25\lambda_R^2 \times \exp (-0.735\sigma_R/T_6)] , \tag{23}$$

$$\lambda_R = 0.246(\rho/\mu_e T_6^{3/2}) \exp (2.515\sigma_R/T_6) .$$

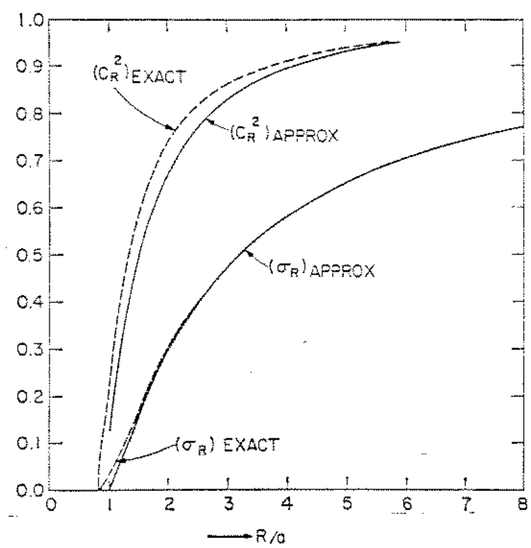

FIG. 1.—Properties of the ground state of the triply ionized Be^7 atom as a function of screening radius R. R is scaled in units of $a = 0.528 \times 10^{-8}$ cm/4 = first Bohr radius of the unscreened triply ionized atom. $C_R^2 = |\psi_R(0)/\psi_1(0)|^2$ and $\sigma_R = -E_R/\chi_1$. Approximate results are obtained with the variational technique.

Values for ω_R/ω_f at various points in the Sears solar model are presented in Table 1. In the inner one-tenth of the Sun's mass, bound-electron capture enhances the $Be^7(e^-,\nu)Li^7$ rate and decreases the calculated B^8 neutrino flux by a factor of about $\frac{5}{4}$. Thus, bound-electron capture has an effect on the solar B^8 neutrino flux which is of the same magnitude as, but which acts in the opposite sense to, the effect due to the recently reported (Parker 1966) increase in the experimentally determined estimate of the $Be^7(p,\gamma)B^8$ cross-section factor.

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