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THE ^{7}Be ELECTRON-CAPTURE RATE

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ABSTRACT

The effect of plasma and bound-electron screening on the continuum capture rate of ^{7}Be$(e^-,\nu)^7$Li is calculated and shown to be negligible for situations in which the proton-proton chain is likely to be important. A more accurate expression for the continuum capture rate is then derived, making use of the numerical work of Bahcall and May. Convenient formulae are presented for use in stellar-model calculations. The average effect of bound-electron capture is also evaluated for several solar models.

I. INTRODUCTION

The predominant reaction by which ^{7}Be is destroyed in the proton-proton chain, under the conditions existing near the center of the Sun, is the electron-capture reaction ^{7}Be$(e^-,\nu)^7$Li. The equilibrium abundance of ^{7}Be is accurately given by equating the rates of ^{3}He$(\alpha,\gamma)^7$Be and ^{7}Be$(e^-,\gamma)^7$Li. The number of ^{7}Be$(p,\gamma)^8$Be reactions occurring per unit of time, which is expected (Bahcall 1964, 1966) to determine the counting rate in experiments designed to detect solar neutrinos (Davis 1964; Davis, Harmer, and Hoffman 1968), is therefore inversely proportional to the ^{7}Be electron-capture rate. Some time ago, Bahcall (1962) computed the ^{7}Be capture rate considering the capture of continuum electrons and neglecting the plasma screening by the ionized gas of the star. More recently, Iben, Kalata, and Schwartz (1967) computed the capture rate of bound electrons in ^{7}Be III and ^{7}Be IV using the Debye-Hückel approximation to estimate the screening effect of the ionized plasma on the rate of bound-electron capture. They found that bound-electron capture increases the total capture rate in the solar interior by 17–25 per cent in the solar interior and that plasma screening strongly affects the rate of bound-electron capture.

We show in § II that the effect of plasma and bound-electron screening is negligible on the *dominant* process of continuum electron capture. We then derive in § III convenient analytic expressions for the total capture rate, making use of the results of Bahcall and May (1968) to calculate more accurately the rate of the continuum process. We also present in § IV the results of some solar-model calculations of the average effect of bound-electron capture on the solar rate of ^{7}Be$(e^-,\nu)^7$Li.

II. SCREENING CALCULATIONS

a) Plasma Screening

We adopt, following Iben *et al.* (1967), the Debye-Hückel approximation for the plasma screening. In this approximation the S-wave component of the continuum electron wave function satisfies the equation (cf. Landau and Lifschitz 1958):

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{4e^2}{r}\exp\left(\frac{-r}{R}\right) - E_R\right]\psi_R(r) = 0\,. \tag{1}$$

Here R is the screening radius, defined by

$$R = [(4\pi e^2/kT)\Sigma_a n_{0a} z_a^2]^{-1/2}\,, \tag{2a}$$

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and n_{0a} is the average number density of the ion species a with charge ez_a. Numerically

$$R \approx 0.298[64T_6/\rho(3+X)]^{1/2}a_0\,, \tag{2b}$$

if the principal constituents of the star are hydrogen and helium. The quantity a_0 is the Bohr radius, T_6 is the temperature in 10^6 °K, ρ is the density in grams per cubic centimeter, and X is the mass abundance of hydrogen. For the temperatures and densities of main interest for the proton-proton chain (i.e., $10 \leq T_6 \leq 17$, and $50 \leq \rho \leq 200$), the radius R satisfies $0.3a_0 \lesssim R \lesssim 1.25a_0$.

We have numerically integrated equation (1) for various values of R in the above range. For $R \geq 0.4a_0$, we found that the difference between the probability density given by the solution of equation (1) and the corresponding solution for the unscreened ($R = \infty$) case is less than 1 per cent. For $0.3a_0 \leq R \leq 0.4a_0$, this difference is less than 2 per cent. We therefore conclude that plasma screening is unimportant for captures from the continuum.

b) Bound-Electron Screening

The potential field in which a continuum electron moves when one electron is bound to the ^{7}Be nucleus can be estimated by solving the following equation:

$$(\nabla^2 - R^{-2})\phi(r) = 4\pi e|\psi_{Rg}(r)|^2 - 16\pi e\delta(r)\,. \tag{3}$$

In equation (3), ψ_{Rg} is the (bound) ground-state solution of equation (1), which includes the effect of plasma screening on the bound electron. The solution to equation (3) may be written in the following form:

$$\begin{aligned} r\phi(r) = 4e\,\exp(-r/R) + 2\pi R\Big\{&\exp(-r/R)\int_0^r dx\,x\rho(x)\,\exp(+x/R) \\ &- \exp(+r/R)\int_0^r dx\,x\rho(x)\,\exp(-x/R) + 2[\sinh(r/R)] \\ &\times \int_0^\infty dx\,x\rho(x)\,\exp(-x/R)\Big\}\,, \end{aligned} \tag{4}$$

where $\rho \equiv -e|\psi_{Rg}(r)|^2$. Equation (4) is convenient when, as is true in the present case, $\rho(x)$ is given only in numerical form.

We have solved numerically equation (1) for the ground-state wave function in a number of cases and have used the results to compute potentials as defined by equation (4). These potentials were then inserted in the Schrödinger equation,

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - e\phi(r) - E_R\right]\psi_R = 0\,, \tag{5}$$

which describes the motion of the continuum electron in the presence of the potential $\phi(r)$. We have solved equation (5) for R in the range $0.3a_0 \leq R \leq 1.25a_0$ and $T_6 \geq 4$.

We find that for all values of R and T in the above range the difference between $|\psi_R(0)|^2$ as calculated from equations (1) and (5) is less than 2 per cent. Hence bound-electron screening is not important for the case of Be IV (one bound electron). The other ionization states of Be are too infrequently occupied to be of importance in stars like the Sun (Iben *et al.* 1967).

We conclude that screening, either by free plasma electrons or by electrons bound to the nucleus, is unimportant for computing the continuum rate of ^{7}Be$(e^-,\nu)^7$Li. Screening is unimportant for continuum electrons, but of major significance for bound electrons, species a with charge ez_a. Numerically

$$+ X)]^{1/2}a_0 \, , \qquad (2b)$$

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because the continuum electrons are of higher energy and spend relatively little time near the Be nucleus about which the plasma electrons are clustered. Detailed calculations were required to estimate reliably how small the screening effect actually is for continuum states.

III. REACTION RATE

The total reaction rate λ for $^7Be(e^-,\nu)^7Li$ can be written

$$\lambda_{total} = \lambda_c[1 + (5.07/T_6) \exp (2.515\sigma_R/T_6)S_R] \, , \qquad (6)$$

where λ_c is the capture rate from the continuum and the bracketed expression represents the additional contribution from bound electrons (Iben *et al.* 1967). The various quantities referring to bound-electron capture have been defined by Iben *et al.* and will be given below in a convenient numerical form.

The continuum capture rate was calculated by Bahcall (1962). A convenient formulation of his result can be given in terms of the integrals $I(\beta)$ defined by equation (29b) of the preceding paper (Bahcall and May 1968). We find:

$$\lambda_c = 4.24 \times 10^{-9}(\rho/\mu_e)T_6^{-1/2}I(4T_6^{-1/2}) \text{ sec}^{-1} \, . \qquad (7)$$

It has been common practice in review articles and in calculations of the solar neutrino flux to simplify Bahcall's result by neglecting some small but complicated terms which are here represented by $[1 - I(4\, T_6^{1/2})]$. We can, however, obtain an accurate approximation for λ_c by making use of the numerical results of Table 1 of the preceding paper. We find

$$\lambda_c = 4.62 \times 10^{-9}(\rho/\mu_e)T_6^{-1/2}[1 + 0.004(T_6 - 16)] \, , \qquad (8)$$

for $10 \leq T_6 \leq 16$. Here μ_e is the mean electron molecular weight ($\approx 2/[1 + X]$). The rate represented by equation (8) is about 9 per cent larger, for the conditions which exist near the center of the Sun, than the rate which has been in current usage (Fowler, Caughlan, and Zimmerman 1967; Bahcall, Bahcall, and Shaviv 1968).

The quantities appearing in the bracketed part of equation (6) are (Iben *et al.* 1967):

$$S_R \equiv C_R^2[1 + 0.435L_R \exp (-0.735\sigma_R/T_6)]/D \, , \qquad (9a)$$

with

$$D \equiv [1 + L_R + 0.25L_R^2 \exp (-0.735\sigma_R/T_6)] \, , \qquad (9b)$$

$$L_R \equiv 0.246(\rho\mu_e^{-1}T_6^{-3/2}) \exp (2.515\sigma_R/T_6) \, . \qquad (9c)$$

Here $C_R = |\psi_{Rg}(0)|/|\psi_{\infty g}(0)|$, where ψ_{Rg} and $\psi_{\infty g}$ are the ground-state solutions of equation (1) with screening radius R and infinity, respectively. The quantity $\sigma_R = E_R/E_\infty$, where E_R and E_∞ are, respectively, the energies corresponding to ψ_{Rg} and $\psi_{\infty g}$. A table of numerical values for σ_R was given by Iben *et al.* (1967). In order to make more convenient the application of equations (6)–(9) to stellar-model calculations, we have derived polynomial representations of C_R^2 and σ_R that are accurate enough to permit the calculation of λ_{total} with an error of less than 3 per cent for all screening radii in the range $0.3a_0 \leq R \leq 1.25a_0$. We find

$$C_R^2 \approx -0.6064 + 4.859r - 5.283r^2 + 1.907r^3 \qquad (10a)$$

and

$$\sigma_R \approx -0.431 + 2.091r - 1.481r^2 + 0.401r^3 \, , \qquad (10b)$$

where $r \equiv (R/a_0)$ (cf. eqs. [2]).

Thus the total reaction rate is given by equation (6) with the continuum capture rate, λ_c, given by equation (8) and the quantities referring to bound capture given by equations (9) and (10).

IV. SOLAR-MODEL RESULTS

The principal application of an accurate expression for the rate of electron capture by ^{7}Be is in calculations of the solar neutrino fluxes. We have therefore made use of calculations carried out on a variety of solar models by N. A. Bahcall (1968) to determine the average increase in the ^{7}Be capture rate due to bound-electron capture. The fractional increase in the total capture rate is represented by the bracketed expression, B, in equation (6): $B = [1 + (5.07/T_6) \exp(2.515\, \sigma_R/T_6) S_R]$. Actually, two different averages are of interest: (i) $\langle B \rangle \equiv [\Sigma_k \phi_k(^7\mathrm{Be}) B_k]/\Sigma \phi_k(^7\mathrm{Be})$, and (ii) $\langle B^{-1} \rangle \equiv [\Sigma_k \phi_k(^8\mathrm{B}) B_k^{-1}]/\Sigma_k \phi_k(^8\mathrm{B})$, where each value of k represents a different spherical shell in the Sun and $\phi_k(^7\mathrm{Be})$, $\phi_k(^8\mathrm{B})$ are, respectively, the ^{7}Be and ^{8}B neutrino fluxes from the kth shell. The first average, (i), represents the effect of bound capture on the rate of nuclear energy generation, and the second average, (ii), represents the influence on the important ^{8}B neutrino flux. The calculations of N. A. Bahcall were carried out on the three solar models labeled C, D, and E by Bahcall, Bahcall, and Shaviv (1968), which represent a probable range for the most likely models, and on a solar model (Bahcall, Bahcall, and Ulrich 1968) in which the Sun was completely mixed for its total lifetime. About sixty-five shells were included for each solar model. For the three likely models C, D, and E, it was found that

$$\begin{aligned} \langle B \rangle &\approx \langle B^{-1} \rangle^{-1} \\ &\approx 1.205 \pm 0.01 . \end{aligned} \tag{11}$$

Thus bound-electron capture increases the total capture rate, in unmixed models, by about 20 per cent. This result is consistent with the somewhat less accurate results of Iben *et al.* (1967) and Bahcall and Shaviv (1968). For the completely mixed model, it was found that $\langle B \rangle \approx \langle B^{-1} \rangle^{-1} \approx 1.25$.

In summary, we note that for almost all solar-model calculations it is sufficient to take as the total ^{7}Be capture rate

$$\lambda_{\mathrm{total}} \approx 1.2 \lambda_c , \tag{12}$$

where λ_c is defined by equation (7). Only 90 per cent (Lauritsen and Ajzenberg-Selove 1966) of the ^{7}Be neutrinos produced with the rate given in equation (13) are above threshold for the reaction $^{37}\mathrm{Cl}(\nu,e^-)^{37}\mathrm{Ar}$, which is used in the neutrino-detection experiment of Davis (1964) and Davis *et al.* (1968).

We are grateful to N. A. Bahcall for providing us with the numerical results giving the average effect of bound-electron captures in several solar models.

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