Geltman's maximum of κ_λ is more accurate than our corresponding maximum given by Eq. (11).

In Table III we have a comparison of our bound-state function φ_0, Eq. (6), with the corresponding function of Geltman. Table III shows that crudely the results are similar. We see that the above given consideration gives just the value of the electron affinity of the hydrogen atom given by Henrich, while the analytical formula obtained for the total continuous absorption coefficient is very simple and its degree of accuracy in comparison with Geltman's results is a good one.

Geltman has simplified extensively the calculation of Chandrasekhar by adopting the cut-off Coulomb potential but the solutions obtained for the discrete and continuous spectrum do not give an analytical expression for the total continuous absorption coefficient.

Concerning Eq. (1) we must make it clear that this equation implies the use of a special form for the bound and continuum two-electron wave functions, as well as the dipole length form of the matrix element. Since the continuum functions used in this paper do not satisfy the Schrödinger equation with $V(r)$ given by Eq. (4), different results will be obtained with the dipole velocity and acceleration forms of the matrix element. A check on the self-consistency of this calculation would be the spread in results with all three forms of the dipole matrix element. The other two could also be obtained analytically without much trouble.

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Theory of Bound-State Beta Decay*

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The theory of beta-decay processes in which an electron is created in a bound atomic state is developed in the allowed approximation. The correlations and total decay rate are calculated with the renormalized $V-A$ theory and the results are valid for atoms of arbitrary electronic configuration. The relative probability of bound-state to continuum-state decay is shown to be independent of nuclear matrix elements; some bound-state decay rates are presented that were calculated by making use of this fact. The possibility of experimentally detecting bound-state decay is also briefly examined. The beta decay of nuclei in stellar interiors is discussed and a number of examples are presented for which bound-state decay is more likely than continuum-state decay under the conditions that obtain in stellar interiors.

I. INTRODUCTION

THE usual theory of beta decay assumes that the transformation of a neutron into a proton is accompanied by the creation, in continuum states, of an electron and an antineutrino. This assumption ignores decays in which an electron is created in a previously unoccupied bound atomic state.

We shall develop, in the allowed approximation, the theory of the usually ignored decays in which a neutron transforms into a proton, an antineutrino is produced in a free state, and an electron is created in a bound atomic state.[1] It is important to realize that the bound-state decay process does not take place through the capture into an atomic orbit of an electron initially created in a continuum state; the direct creation of an electron in a bound state is more probable than the capture process.

The relative frequency of bound-state to continuum-state decays can be estimated with a phase-space argument that does not depend on the formal theory of weak interactions. The phase-space volume available for continuum state decays is represented by the function $f(Z,W_0)$,[2] where the dependence on Z indicates that the Coulomb correction has been included. For bound-state decays, the analogous corrected phase space volume is the square of the neutrino's momentum times the square of the modulus of the electron's wave function evaluated at the nuclear surface. The relative frequency of bound-state to continuum-state decays is approximately equal to the ratio of the phase-space volumes when these volumes are corrected for the electron density at the nuclear surface. Thus the relative frequency of bound-state to continuum-state decays is

$$\begin{aligned}\Gamma_B/\Gamma_C &\sim q^2|\Psi(R)|^2/f(Z,W_0)\\ &\sim (W_0-1)^2(\alpha Z)^2/n^3 f(Z,W_0).\end{aligned} \quad (1)$$

This ratio depends sensitively on the nuclear energy release, W_0; the atomic number, Z, of the daughter nucleus; and the principal quantum number, n, of the lowest unoccupied atomic orbit.

The possibility of bound-state decays was first

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[1] In terrestrial experiments, the daughter atoms are almost always neutral and hence difficult to detect.

[2] We use here units in which $\hbar=m=c=1$. The function f is most familiar in the combination $\log ft$.

pointed out by Daudel *et al.*,[3] in 1947. They examined, with the original Fermi (vector) theory, the probability of creating an electron in the K shell and suggested that bound-state decay would be most important in the stars. Their work properly applies only to completely ionized atoms.[4] The theory of Daudel *et al.* was used by Sherk[5] to evaluate the bound-state decay probability for tritium and recently Galzenati *et al.*[6] have calculated the bound-state decay rate for the neutron.

The present treatment applies to atoms of arbitrary electronic configuration and includes the possibility of decay into excited states of the final atom. We use, in Sec. II, the renormalized $V-A$ theory to determine the various correlations and the total decay rate. We present, in Sec. III, some simple examples and examine the possibility of detecting bound-state decays. In Sec. IV, we discuss the beta decay of nuclei in stellar interiors and show that for some nuclei bound-state decay is more likely than continuum-state decay under the conditions that obtain in stellar interiors.

II. GENERAL THEORY

We consider an initial state consisting of nucleus with charge $Z-1$ surrounded by N atomic electrons, and represent this state by the vector

$$|i\rangle = |Z-1, N, \gamma; k\rangle.$$

We make no restriction on the form of the representative of this state vector; the electron and nuclear states will be specified by the two sets of quantum numbers γ and k, respectively. The final electron states will be described by a complete set of eigenstates of an appropriate Hamiltonian, which, in practice, will usually be the Dirac Hamiltonian with a nuclear field $-Ze^2/r$. The final states of the system will be represented by

$$|f\rangle = |Z, N+1, \gamma'; \bar{\nu}; k'\rangle,$$

where $\bar{\nu}$ specifies the properties of the antineutrino that is created.

The weak interaction producing the decay is assumed to be

$$H_w = (G_V/\sqrt{2})(\bar{\psi}_p\gamma_\alpha(1-x\gamma_5)\psi_n) \times (\bar{\psi}_e\gamma_\alpha(1+\gamma_5)\psi_\nu) + \text{H.c.}, \quad (2)$$

where

$$x \equiv C_A/C_V.$$

We write

$$\bar{\psi}_e = \textstyle\sum_b a_b^\dagger \bar{\varphi}_b + \sum_c a_c^\dagger \bar{\varphi}_c + \text{positron operators}, \quad (3)$$

where $a_b^\dagger$ and $a_c^\dagger$ create electrons in bound and continuum states, respectively. In the usual theory of beta decay, the four-component spinors $\bar{\phi}_c$ are chosen to be continuum eigenfunctions of the Dirac Hamiltonian for an electron in the Coulomb field of a nucleus of charge Z.[7] The necessity for including the bound-state operators in (3) becomes clear when the incompleteness of the continuum Coulomb eigenfunctions is considered.

If the bound-state decay rate is calculated in the usual way, one must evaluate

$$\textstyle\sum_b \bar{\phi}_b \langle Z, N+1, \gamma' | a_b^\dagger | Z, N, \gamma\rangle. \quad (4)$$

There will in general be an infinite number of nonvanishing terms in this summation due to the lack of orthogonality between the eigenfunctions of the initial and final Hamiltonians.[8] This difficulty can be avoided by expanding the initial electron state in terms of eigenstates of the final Hamiltonian. The bound-state decay rate can then be written

$$\begin{aligned}\Gamma = \frac{2\pi}{\hbar}\sum_i \sum_f \sum_{\gamma'',\gamma'''}{}^{*} &\langle Z, N, \gamma'' | Z-1, N, \gamma\rangle^* \\ &\times \langle Z, N, \gamma''' | Z-1, N, \gamma\rangle \\ &\times \langle Z, N+1, \gamma'; \bar{\nu}; k' | H_w | Z, N, \gamma''\rangle^* \\ &\times \langle Z, N+1, \gamma'; \bar{\nu}; k' | H_w | Z, N, \gamma'''\rangle \\ &\times \delta(E(\gamma''; k) - E(\gamma'; \bar{\nu}; k')). \quad (5)\end{aligned}$$

The asterisk on the summation over γ'' and γ''' indicates that only those states for which

$$E(\gamma'''; k) = E(\gamma''; k) \quad (6)$$

are to be included.

It will usually be convenient to describe the final electron states by the complete set of Slater determinants (rank $N+1$) formed from the one-particle solutions of the Dirac equation with an external Coulomb field $-Ze^2/r$. With this choice of final Hamiltonian and basis vectors, the cross terms in (5) arise from the energy degeneracy of the one-electron Dirac eigenfunctions with respect to the spin-orbit quantum number[9]

$$\kappa = \begin{cases} l & \text{for } l = j+\frac{1}{2} \\ -l-1 & \text{for } l = j-\frac{1}{2}. \end{cases} \quad (7)$$

In Eq. (7), l is the orbital angular momentum associated with the large component of the Dirac spinor. We make the usual assumptions of allowed beta decay: (1) The leptonic current is evaluated at the nuclear surface;

[3] R. Daudel, M. Jean, and M. Lecoin, J. phys. radium **8**, 238 (1947); Compt. rend. **225**, 290 (1948); R. Daudel, P. Benoist, R. Jacques, and M. Jean, *ibid.* **224**, 1427 (1947).

[4] See Secs. II and III for a discussion of this point.

[5] P. M. Sherk, Phys. Rev. **75**, 789 (1949).

[6] E. Galzenati, M. Marinaro, and S. Okubu, Nuovo cimento **15**, 934 (1960).

[7] In order to include screening corrections, an average field due to the atomic electrons is sometimes used. See J. R. Reitz, Phys. Rev. **77**, 10 (1950) and references cited therein.

[8] One can show that this lack of orthogonality causes no complications in the pure continuum-state theory of beta decay.

[9] M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons, Inc. New York, 1961), p. 158.

and (2) the nucleons are treated nonrelativistically. Inserting the definitions of $\bar{\psi}_e$ and H_w in Eq. (5), the evaluation of the decay rate can be carried out in the usual way. We obtain:

$$\Gamma_B = G_V^2 \frac{(\alpha Z)^3 (mc)^5}{8\pi^3 \hbar^7 c} \left(\frac{W_0}{mc^2} - 1\right)^2 \sum_i \sum_{\gamma'',\gamma'''}^{*} \times d\Omega_q \langle Z, N, \gamma'' | Z-1, N, \gamma\rangle^* \langle Z, N, \gamma''' | Z-1, N, \gamma\rangle \times \left(1 + \frac{b}{W_0 - mc^2}\right)^2 \pi \left(\frac{a_0}{Z}\right)^3 \times \phi^\dagger_{\gamma',\gamma'''}(R) \left[A + \left(\frac{M}{I}\right) B + \left(\frac{M^2 - \frac{1}{3}I(I+1)}{I(2I-1)}\right) C\right] \times (1+\gamma_5)\phi_{\gamma',\gamma''}(R), \quad (8)$$

where b = binding energy of created electron, $\hat{I}$ = unit vector in the direction of the initial nuclear spin, M = projection of the initial nuclear spin, m = mass of the electron, R = nuclear radius, W_0 = nuclear energy release, and[10]

$$A = \delta_{I,I'} \langle 1\rangle^2 (1 - \boldsymbol{\sigma}\cdot\hat{q}) + x^2 (1 + \tfrac{1}{3}\boldsymbol{\sigma}\cdot\hat{q}),$$

$$B = x^2 \langle M_s\rangle \langle\sigma\rangle^2 (\boldsymbol{\sigma} + \hat{q})\cdot\hat{I} + 2\delta_{I,I'} x \langle 1\rangle\langle\sigma\rangle \times (I/I+1)^{\frac{1}{2}} (\boldsymbol{\sigma} - \hat{q})\cdot\hat{I}, \quad (10)$$

$$C = x^2 \langle\sigma\rangle^2 \Lambda (3\hat{q}\cdot\hat{I}\boldsymbol{\sigma}\cdot\hat{I} - \hat{q}\cdot\boldsymbol{\sigma}).$$

The spinor wave function $\phi_{\gamma',\gamma''}$ represents the electron that must be added to $|Z, N, \gamma'\rangle$ in order to form the state $|Z, N+1, \gamma'\rangle$; the value of $\phi(R)$ is given in the appendix. The quantities $\langle M_S\rangle$ and Λ are the usual parameters that occur in the literature; they are also defined in the Appendix. The binding energy b is retained in Eq. (8) because it is important for heavy completely ionized atoms which can exist in stellar interiors.

If we choose the Dirac Coulomb Hamiltonian as the zeroth-order final-state Hamiltonian, the only interference terms that survive the transition from Eq. (5) to Eq. (8) are terms in which γ'' and γ''' differ only by the sign of κ for the created electron. This interference contribution has little practical effect, since for $\gamma'' \neq \gamma'''$ the expression

$$\phi_{\gamma',\gamma''}{}^\dagger(R) \left[A + \left(\frac{M}{I}\right) B + \left(\frac{M^2 - \frac{1}{3}I(I+1)}{I(2I-1)}\right) C\right] \times (1+\gamma_5)\phi_{\gamma',\gamma'''}(R) \quad (11)$$

is pure imaginary.[11] By correctly choosing the phases of the basis vectors, the main contribution of the interference terms can be shown to be zero.

The usual allowed selection rules are manifest in Eqs. (8) and (10) and the correlation coefficients have a familiar form. For initially unpolarized nuclei, only the term involving A is present. The total bound-state decay rate for experiments in which no attempt is made to observe the direction of the nuclear recoil and in which the nuclei are initially unoriented is

$$\Gamma_B = \frac{G_V^2 (\alpha Z)^3 (mc)^5}{2\pi^2 \hbar^7 c} \left(\frac{W_0}{mc^2} - 1\right)^2 \zeta \Sigma, \quad (12)$$

where

$$\zeta = \delta_{I,I'} \langle 1\rangle^2 + x^2 \langle\sigma\rangle^2, \quad (13)$$

and

$$\Sigma = \sum_{\gamma'} \sum_{\gamma''\,\gamma'''}^{*} \langle Z, N, \gamma'' | Z-1, N, \gamma\rangle^* \times \langle Z, N, \gamma''' | Z-1, N, \gamma\rangle \left(1 + \frac{b}{W_0 - mc^2}\right)^2 \times \pi \left(\frac{a_0}{Z}\right)^3 \phi_{\gamma',\gamma'''}{}^\dagger(R)(1+\gamma_5)\phi_{\gamma',\gamma''}(R). \quad (14)$$

The ratio of the number of bound-state to the number of continuum-state decays is therefore

$$\frac{\Gamma_B}{\Gamma_C} = \frac{\pi(\alpha Z)^3}{f(Z, W_0)} \left(\frac{W_0}{mc^2} - 1\right)^2 \Sigma. \quad (15)$$

This ratio is independent of nuclear matrix elements and of the value of $x = C_A/C_V$. This independence of nuclear parameters enables us to calculate the bound-state decay rate of a particular nucleus provided we know empirically its continuum-state decay rate. The quantities that are most important in determining the magnitude of the ratio Γ_B/Γ_C occur as factors multiplying Σ in Eq. (15); these factors are identical with the ones found by the phase-space argument given in the Introduction.

Note added in proof. The interference between terms having different values for the sign of K can be eliminated by assuming that atomic ground states are nondegenerate and using parity arguments. This simplifies Eq. (14) if the Dirac Coulomb Hamiltonian is chosen as the zeroth-order Hamiltonian.

III. SIMPLE EXAMPLES

The results of the preceding section are most simply applied to atoms that initially possess no bound electrons. For such atoms, Σ can be written

$$\Sigma = \sum_{\gamma'} \left(1 + \frac{b}{W_0 - mc^2}\right)^2 \pi \left(\frac{a_0}{Z}\right)^3 \phi_{\gamma'}{}^\dagger(R)\phi_{\gamma'}(R). \quad (16)$$

[10] We use the definition of reduced matrix elements that has been suggested by E. J. Konopinski:

$$\langle I'M' | S_{Jm}{}^\dagger | IM\rangle = \langle I'M'Jm | IM\rangle \langle S_J\rangle.$$

This definition simplifies certain formulae in the theory of beta-decay. In the spinor matrix element (8), γ_5 is $-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; it contributes only if κ''' is different from κ'', in which case the one in $(1+\gamma_5)$ does not contribute.

[11] See Appendix I for a proof of this fact.

This simplified form of Σ can be used for many astrophysical applications.

Equations (15) and (16) can also be used to calculate the relative number of times a neutron decays by emitting a neutral hydrogen atom and an antineutrino instead of emitting a proton, a free electron, and an antineutrino. We find

$$\Gamma_B/\Gamma_C=4.2\times10^{-6}, \tag{17}$$

where 20% of the bound-state decay probability is due to decays into the excited states of the hydrogen atom.[12]

The most favorable case for laboratory detection of bound-state decay is the $H^3\rightarrow He^3$ transition for which $W_1-mc^2=0.035\ mc^2$ and $f=2.8\times10^{-6}$. The tritium bound-state decay rate is easy to calculate since nonrelativistic wave functions are adequate to determine the decay probability to two parts in 10^4 $[(\alpha Z)^2]$. One can derive a general formula for the overlap integrals $\langle Z=2,\ n,\ \kappa=-1|Z=1,\ n=1,\ \kappa=-1\rangle$ and, therefore, Σ can be computed as accurately as desired. We obtain

$$\begin{aligned}\Gamma_B/\Gamma_C&=4.3\times10^{-3}\Sigma\\&=6.9\times10^{-3}.\end{aligned} \tag{18}$$

Excited states contribute about 22% to this ratio.

A recent series of ingenious experiments performed at Oak Ridge National Laboratory[13] suggests a possible approach to the problem of detecting bound-state beta decay. The Oak Ridge group used a mass spectrometer in combination with electric and magnetic fields to measure the charge on daughter nuclei from the He^6 to Li^6 decay. A direct utilization of their experimental arrangement is not possible since an electric field was used to remove the charged nuclei from the region of decay, but future modifications may make possible the detection of bound-state decay.

The ratio (18) has also been computed by Sherk[5] who used the original theory of Daudel *et al.*[3] Since the original theory did not take into account either the nonorthogonality of initial and final electron eigenfunctions [Eq. (4)] or the contribution of excited states, our results differ appreciably from those of Sherk.[14] Moreover, we have ignored the final state electrostatic interaction between the two electrons in the He^3 atom, since one can show that the ratio Γ_B/Γ_C is independent of the choice of basis functions if the electron's binding energy is neglected. Sherk, following the suggestion of Daudel *et al.*, took account of the final state electron-electron interaction by using a screened hydrogenic wave function. He determined the screening parameter by minimizing the total energy. The wave function found in this way has a $Z_{\rm eff}$ of 1.688, which yields an electron density at the origin of about 0.6 the unscreened value. This screening method is incorrect since it over-emphasizes the region of space in which the two electrons are close together.[15]

TABLE I. The relative probability of bound-state to continuum-state decay for some nuclei when assumed completely ionized. The ratios were calculated using nonrelativistic wave functions.

Isotope	W_0 (units of mc^2)	$\mathrm{Log}_{10}f(Z,W_0)$	Γ_B/Γ_C
${}_6C^{14}$	1.31	−2.25	0.01
${}_{14}Si^{32}$	1.20	−2.65	0.1
${}_{28}Ni^{63}$	1.13	−2.9	0.9
${}_{41}Nb^{95}$	1.29	−1.5	0.7
${}_{44}Ru^{106}$	1.08	−3.28	7
${}_{46}Pd^{172}$	1.55	−0.60	0.3
${}_{47}Ag^{110}$	1.17	−2.1	2
${}_{63}Eu^{155}$	1.30	−1.0	1
${}_{76}Os^{191}$	1.28	−0.85	1

IV. ASTROPHYSICAL APPLICATIONS

The current theories of element formation in stars[16,17] assume that the appropriate nuclear reactions occur at very high temperatures, consistent with models of stellar structure and evolution. The temperatures assumed are so high that even the heavier nuclei are stripped of electrons, and thus bound-state decay to the K shell is possible for heavy nuclei. In the calculation of the relative abundances of elements produced by nuclear processes in the interior of stars, the decay rates that are expected for ionized atoms should be used. These calculated decay rates can differ by orders of magnitude from the rates measured under normal terrestrial conditions.[18]

Several examples of the ratio Γ_B/Γ_C are given in Table I for completely ionized atoms. Nonrelativistic wave functions were used in the calculations and are

[12] Our result (17), even ignoring the contribution of excited states, differs by a factor of ζ (neutron)/4 from the recent calculation of E. Galzenati *et al.* cited in reference 6. These authors used an experimental value for Γ_C and a theoretical value for Γ_B that was calculated assuming $x=-1$. Hence, they did not take advantage of the fact that Γ_B/Γ_C is independent of nuclear matrix elements. Galzenati *et al.* also discuss the probability of bound-state decay for Λ and K particles.

[13] T. A. Carlson, C. H. Johnson, and Frances Pleasonton, Bull. Am. Phys. Soc. **6**, 227 (1961). I am grateful to Dr. Carlson for preprints of their work and for an informative private communication on the experimental detection of bound-state decay.

[14] The numerical disagreement is not large since the effect of the several calculational differences is partially cancelled by the fact that Sherk used too low a value for the decay energy, W_0.

[15] A more correct procedure is to modify the hydrogenic Schrödinger equation for $Z=2$ by adding the potential due to an electron in the 1s orbit of H^3. This can be done by assuming $\rho(r)=-e|\psi(r)|^2$.

[16] E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Revs. Modern Phys. **29**, 547 (1957).

[17] A. G. W. Cameron, Ann. Rev. Nuclear Sci. **8**, 299 (1958).

[18] The decay rates for completely ionized Pb^{210} and Pu^{241} differ by three orders of magnitude or more from their normal terrestrial values. These nuclei were not included in Table I because their decays are probably first parity forbidden and because relativistic effects are important for such high Z. In calculating the decay rate for nuclei with such high Z, it is important to realize that the maximum beta-ray energy that is measured in the laboratory is $W_0-mc^2+\Delta B$, where ΔB is the difference in binding energies between initial and final atomic states. The quantity ΔB is much larger than W_0-mc^2 for Pb^{210} and Pu^{212}. See, for example, M. S. Freedman, F. Wagner, Jr., and D. W. Engelkemeir, Phys. Rev. **88**, 1155 (1951).

accurate to terms of order $(\alpha Z)^2$.[19] The case of Ru^{106} is instructive since the half-life for a completely ionized Ru^{106} atom is 8 times shorter than for a neutral atom. The decay of Cs^{134} via two beta groups, which have maximum energies of 0.086 Mev and 0.652 Mev, gives rise to an interesting phenomenon that is not illustrated in Table I. The Cs^{134} decay occurs via the higher energy transition in 80% of the terrestrial decays, but the lower and higher energy transitions occur about equally often if the Cs^{134} atom is stripped of electrons. A similar situation obtains for Te^{132}.

The isotope Sm^{151} is particularly interesting since the half-life of this isotope has been used to determine the time scale for the slow neutron capture process (s process) in the region $63 \leq A \leq 209$.[16] This estimate is somewhat in error since the terrestrially measured half-life was used; the ratio Γ_B/Γ_C obtained from Eqs. (15) and (16) for Sm^{151} is 3. It is likely, however, that Sm^{151} is first parity-forbidden, in which case one cannot apply Eqs. (15) and (16) without further justification. There are a number of other first parity-forbidden transitions for which the ratio Γ_B/Γ_C obtained from Eqs. (15) and (16) is large, for example, Ce^{144}, Pm^{147}, Tm^{171}, Au^{199}, Hg^{203}, and Pb^{212}.[18] The author is currently investigating the bound-state decay probability for forbidden transitions.

V. CONCLUSION

We have developed the allowed theory of bound-state decay for atoms of arbitrary electronic configuration and have calculated the probability of bound-state decay for some simple examples by making use of the fact that the relative probability of bound-state to continuum-state decay is independent of nuclear matrix elements. We have also demonstrated the importance of bound-state decay for nuclei in stellar interiors.

[19] We have also neglected the difference between the maximum beta-ray energy measured in the laboratory and $W_0 - mc^2$. This approximation is valid for the cases we consider in Table I but it is not always justified.

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APPENDIX

The wave functions $\phi_{n,-1}(R)$ are of the form[9]

$$\phi_{n,-1}(R)=\begin{pmatrix} g\chi_{-1}^{\mu} \\ +if\chi_{+1}^{\mu}\end{pmatrix} \cong (1+c\boldsymbol{\alpha}\cdot\hat{r})\begin{pmatrix} g\chi_{-1}^{\mu} \\ 0\end{pmatrix}, \quad \text{(A1)}$$

where

$$c=-if(R)/g(R).$$

We have used the fact that f is proportional to g for small r and that

$$\boldsymbol{\sigma}\cdot\hat{r}\chi_{\kappa}^{\mu}=-\chi_{-\kappa}^{\mu}.$$

The explicit forms of the real functions f and g are given in reference (9). The part of $\phi(R)$ that involves α does not contribute to the allowed decay rate. Thus

$$\phi_{n,-1}(R)\cong\begin{pmatrix} g\chi_{-1}^{\mu} \\ 0\end{pmatrix}, \quad \text{(A2)}$$

and

$$\phi_{n,+1}(R)\cong\begin{pmatrix} 0 \\ if\chi_{-1}^{\mu}\end{pmatrix}. \quad \text{(A3)}$$

It is easy to see that expression (11) is pure imaginary if the forms (A2) and (A3) are used for the wave functions.

The quantities $\langle M_S\rangle$ and Λ are defined as follows:

$$\langle M_S\rangle=\tfrac{1}{2}[I(I+1)-I'(I'+1)+2]/(I+1). \quad \text{(A4)}$$

and

I'	$I-1$	I	$I+1$
Λ	1	$\dfrac{-(2I-1)}{(I+1)}$	$\dfrac{I(2I-1)}{(I+1)(2I+3)}$

(A5)