Branching ratio in the decay of ^{7}Be

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The branching ratio for ^{7}Be electron-capture decay to the first excited state in ^{7}Li has been measured by implanting a 20-MeV ^{7}Be beam into a silicon detector telescope and counting the subsequent γ decays with well calibrated Ge(Li) detectors. A branching ratio of $10.7\pm0.2\,\%$ was obtained. This value is in agreement with past measurements but does not agree with a recently suggested higher value. Sources of uncertainties and implications for nuclear physics and astrophysics are discussed.

[RADIOACTIVITY ^{7}Be; measured the EC branching ratio to the ^{7}Li first excited state.]

I. INTRODUCTION

The electron-capture decay of ^{7}Be proceeds mostly to the $\frac{3}{2}^-$ ground state of ^{7}Li but includes a significant branch to the $\frac{1}{2}^-$ first excited state at 477.61 keV. The branching ratio between these two decays is an important quantity in nuclear physics, both theoretically (as a constraint on the p-shell configuration mixing for these states[1]) and experimentally (since the γ decay from the branching to the first excited state has often been utilized[2–4] to normalize the cross sections for reactions which lead to the production of ^{7}Be). In the latter context, the branching ratio of the ^{7}Be has recently become intimately connected[5–15] with the long standing discrepancy between the apparent observed flux of energetic solar neutrinos[16] and the flux predicted theoretically.[17] The ^{3}He$(\alpha,\gamma)^7$Be reaction is an important link in the chain of reactions which lead to the production of observable solar neutrinos. It has been proposed[6,7] that the discrepancy between a recent measurement[18] of this reaction rate and other measurements[4,5,19,20] may relate to the value for the ^{7}Be branching ratio used to normalize the ^{3}He$(\alpha,\gamma)^7$Be reaction cross section. A value of $15.4\pm0.8\,\%$ has been suggested[6]. This represents a substantial increase over the value of $10.39\pm0.06\,\%$ obtained from a weighted average of previous measurements,[21] and is inconsistent with other recent measurements.[8–15] If the much larger value for the branching ratio were correct, it would have a significant effect on the solar neutrino problem and other cross section measurements and theoretical studies as well.

In view of the importance of this quantity an independent measurement of the ^{7}Be branching ratio was made. The measurement reported here relies on the implantation of ^{7}Be ions into a silicon surface-barrier detector and in that sense is similar to the technique used in Ref. 7. However, we utilize a ^{7}Be beam which is produced, isolated, and focused in a quadrupole sextuplet beam transport system (QSBTS) to implant energetic ($\sim$20 MeV) ^{7}Be ions into the silicon detector. This spectrometer system has a 3 m separation between the target and detector and can produce a well-defined ^{7}Be beam. Furthermore, the utilization of energetic ^{7}Be ions minimizes uncertainties due to straggling. To ensure the reliability of ^{7}Be production and the detection of the absolute flux of the 478 keV γ rays, three different detectors were implanted with ^{7}Be ions and counted with two different, independently calibrated high-resolution Ge(Li) detectors. We obtain a weighted average value for the branching ratio of $10.7\pm0.2\,\%$ which agrees with the previously established number. In Sec. II the details of the technique are discussed. In Sec. III the results are presented. In Sec. IV the theoretical significance of the ^{7}Be branching ratio is also discussed.

II. THE TECHNIQUE

A. ^{7}Be production and detection

The most important component in this step of the measurement is the QSBTS which has been designed[22] for experiments with radioactive ion beams. A schematic drawing of this system is shown in Fig. 1.

In the present experiment a 24-MeV ^{7}Li^{3+} beam from the LLNL tandem Van de Graaff facility impinged on a thin (1–2 mg/cm^2) polyethylene target, $(CH_2)_n$. Most of the incident beam then struck a 0° shadow bar which also served as a split Faraday cup to monitor beam intensity and position. The emerging 20-MeV ^{7}Li and ^{7}Be^{4+} ions [produced mostly by ^{1}H(^{7}Li,^{7}Be)n reactions] were transported and focused through a series of six quadrupole lenses. After passage through a 5 mm diam collimator, the beam finally impinged on a solid-state detector telescope consisting of a 6.1 μm thick ΔE detector and a 400 or 1500 μm thick E detector. The E detectors were much thicker than the range of 20-MeV ^{7}Be ions ($\sim$60 μm Si). All impinging ^{7}Be ions which passed through the collimator were within the 50 mm^2 active area of the detectors. The two parameter ΔE-E spectrum allowed for the ^{7}Be ions to be easily separated from other possible contaminants and gave a clear picture of the ^{7}Be energy spectrum. To ensure that all events were counted, no two-parameter coincidence was required in the ΔE-E spectrum. Instead, the two dimensional spectrum was only gated by the E-detector signal. The ^{7}Be count rate was maintained at between 500–2000 sec^{-1} during different runs with different thickness targets by maintaining an incident ^{7}Li^{3+}

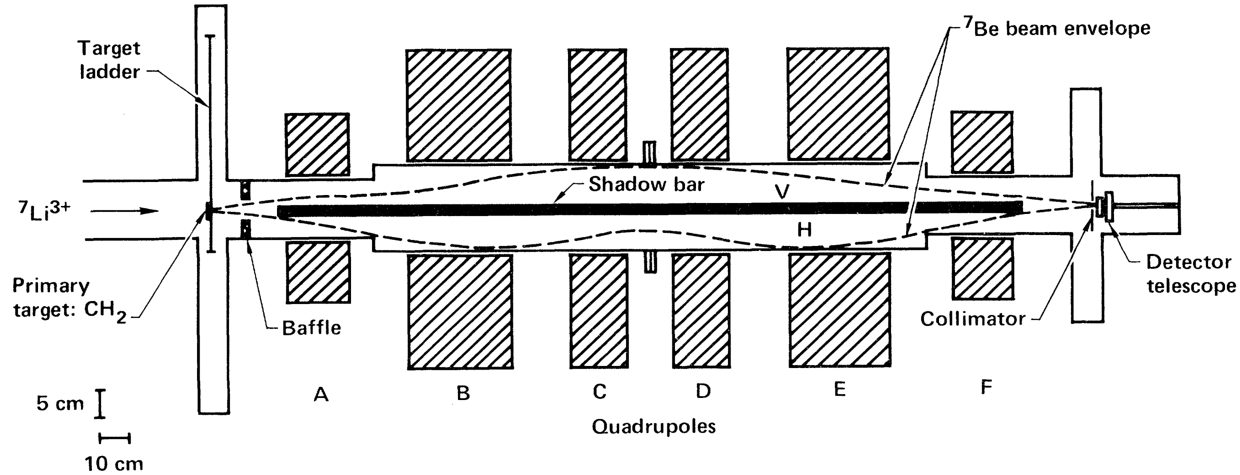

FIG. 1. The quadrupole sextuplet beam transport system (QSBTS) for producing a focused and collimated beam of $^7Be^{4+}$ ions.

current to 1.5 to 3.0 nA.

Figure 2 shows a typical ΔE vs E map for events in the detector telescope. Most of the scattered ^{7}Li was stopped by the shadow bar or the spectrometer walls. Nevertheless, $^7Li^{3+}$ ions inelastically scattered to 11 MeV can have approximately the same trajectory through the spectrometer as the 20 MeV $^7Be^{4+}$ ions. Therefore, in addition to the ^{7}Be peak in Fig. 2 (which constitutes $\sim 60\%$ of the total events), there is a lower energy peak due to 11-MeV

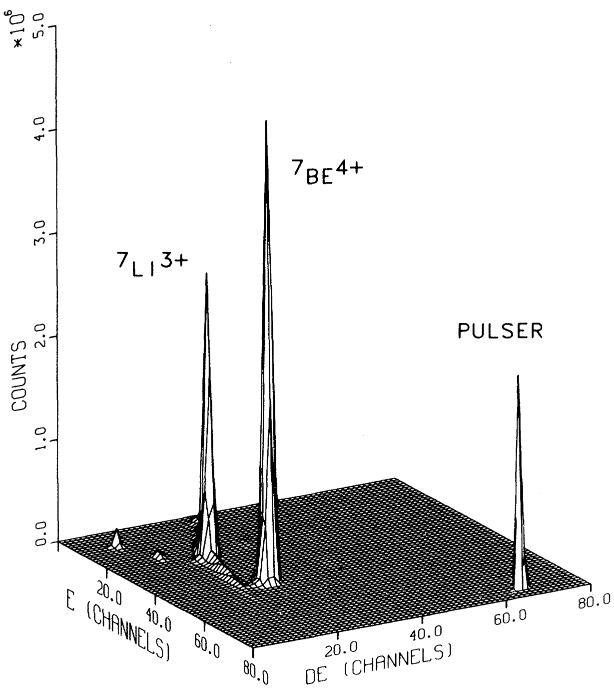

FIG. 2. ΔE vs E contours of the ^{7}Be beam implanted into the E detector.

$^7Li^{3+}$. Other heavy contaminants and pileup events, barely identifiable in Fig. 2, contribute less than 1% to the total collected events and are easily separated from the ^{7}Be contour. It is clear from Fig. 2 that there are very few lower-energy ^{7}Be ions and hence it is unlikely that there are below-threshold undetected ^{7}Be ions implanted in the detector. If such undetected ions were present they could lead to an erroneously high value for the branching ratio.

A pulser peak at high E and ΔE (triggered by the ^{7}Be peak) was used to monitor the total system dead time by comparison with a real-time scaler. The total dead time correction was typically $<2\%$ and consistently maintained at less than 5% during the runs. As an independent check on the dead time corrections the raw ΔE and E spectra were also accumulated (without the requirement of a gate signal generated by the E detector). It was found that no significant dead time was introduced by the gating requirement.

Three separate runs lasting for 15, 41, and 42 h were performed. The detectors (labeled A, B, and C in Table I) were implanted with 2.7×10^7, 8.0×10^7, and 15×10^7 ^{7}Be ions, respectively, before correction for ^{7}Be decay. The total yield was corrected for ^{7}Be decay during the irradiation according to the numerical solution to the integral

$$^7\mathrm{Be}(t)=e^{-\lambda t}\int_0^t R(t')e^{\lambda t'}dt' , \quad (1)$$

where $R(t)$ is the rate of ^{7}Be production (averaged over one hour intervals during the run) and λ is the ^{7}Be decay rate (taken here to be ln2/53.29 d^{-1}).[21] These corrected yields are summarized in Table I.

B. Measurement of the gamma rays

Prior to implantation, the silicon E detectors were observed with Ge(Li) counters to ensure that the detectors were free of ^{7}Be contamination from previous usage. One detector (A) which had been used in a preceding ^{7}Be experiment was observed to have a background of $\sim$ 1.1$\pm$0.6 counts/min of ^{7}Be activity. This is corrected for in Table I and contributes an uncertainty of $<3\%$ to the final

TABLE I. Summary of branching ratio measurements.

	^{7}Be production		Gamma count						
Detector	Length of irradiation (h)	^{7}Be ions at end of irradiation[a] ($t=0$)	Ge(Li) system	Distance to detector (cm)	Start time (d)	Stop time (d)	478 keV photopeak area[b]	Ge(Li) efficiency[c]	Branching ratio
A	14.9	$2.598(0.012)\times10^7$	2	0.53	1.0290	1.9904	1057.(58.)	0.0315(0.0010)	10.5±0.6%
B	40.7	$7.671(0.009)\times10^7$	2	4.35	20.0159	33.6077	6160.(170.)	0.006 20(0.000 10)	10.4±0.3%
C	42.0	$12.46(0.21)\times10^7$	1	6.35	17.8002	31.1073	4600.(140.)	0.002 74(0.000 04)	10.7±0.4%
			1	1.89	42.1903	50.4111	9750(150.)	0.0120(0.0002)	11.1±0.3%
Weighted average[d]									10.7±0.2%

[a]Uncertainty in parentheses is mostly due to identification of ^{7}Be boundaries in the ΔE-E map.
[b]Statistical uncertainty and uncertainty due to background subtraction.
[c]Uncertainty due to counting geometry and calibration source strength.
[d]Uncertainty is quadrature sum of weighted statistical and systematic uncertainties.

branching ratio measured with this detector. The other detectors showed no contamination with an upper limit of $<0.1\%$ for an uncertainty in the branching ratio due to the background correction.

The γ decay of the implanted ^{7}Be ions was independently measured using two of a series of shielded Ge(Li) detector systems (indicated as systems 1 and 2, in Table I) which are routinely used to measure absolute disintegration rates from environmental samples and debris from underground nuclear explosions. These systems have been in operation for several years and have an historical record of precision reproducibility of absolute intensity measurements to better than $\pm2\%$. For the present measurements, the existing efficiency calibration for each system was checked against standard radioactive sources placed at a distance of ~18 cm from the detector housing. A detailed example of a comparison of the measured and quoted decay rates from a series of selected standard sources is noted in Table II. The sources particularly check the efficiency calibration around the ^{7}Be 477.6-keV decay line. The results suggest a standard deviation of $\pm1.4\%$ in reproducing the standard source strengths. We take this standard deviation as the systematic uncertainty due to calibration source uncertainty.

Since the γ-radiation flux from the implanted ^{7}Be sources was relatively weak, a closer counting geometry (<18 cm) had to be employed to obtain reasonable statistics. In this mode, the finite size of both source and Ge(Li) detector are important parameters and must be taken into account. By using a model[23,24] which reduces both the source and the detector to geometric points, the geometric corrections could be made and the uncertainties due to these corrections reliably estimated. The geometric correction was checked by using a small quantity of chemically separated ^{7}Be deposited on a silicon detector. The source material was deposited as a thin film (<0.0015 cm thick) over an area of ~0.5 cm in diameter to reasonably approximate the spot size of the implanted ^{7}Be sources. The γ radiation from the chemically separated source was subsequently measured in each spectrometer system at precisely known positions (±0.01 cm relative to the detector housing) and the geometric correction was applied. A summary of these data for one of the spectrometer systems (2) is shown in Table III. The data show that for source-to-detector separations between ~18 and 2 cm, the model yields a constant source strength to better than $\pm0.5\%$. This indicates the accuracy to which the geometric correction can be made.

Counting times for each implanted ^{7}Be source ranged from 1 to 14 d. Prior to each counting experiment a ^{57}Co radiation source was placed near the detector housing to yield a counting rate of ~80 sec^{-1} for the 121-keV decay line. This source remained fixed through all counts for calibration and background as well as throughout the count of the implanted ^{7}Be detectors. During periods of extended counting, the accumulation rate in the 121-keV photopeak was checked periodically for any deviations in the counting rate due to system instabilities. An additional check was done by measuring the decay rate of a standard ^{137}Cs source several times before and after each long count. These data showed no detectable changes ($\leq0.5\%$) in the detector efficiency during the extended counts. The overall system dead time was determined primarily by the

TABLE II. Selected comparison of measured and quoted absolute standard γ-ray source strengths.

Nuclide	ID #	Quoted source strength (dpm×10^6)[a]	Measured source strength (dpm×10^6)[b]	Δ (%)	Vendor[c]
		System[d] 1			
^{133}Ba	3432	2.597(0.2%)	2.612(1.5%)	−0.5	LMR
^{137}Cs	1901	2.424(1.5%)	2.476(1.1%)	−2.1	LMR
^{137}Cs	070	22.80(1.3%)	22.85(1.0%)	−0.2	IAEA
^{137}Cs	74-092	22.64(1.7%)	22.61(1.0%)	−0.1	OMH
^{137}Cs	3962	24.80(1.3%)	25.29(2.6%)	+2.0	AMR
		System[d] 2			
^{133}Ba	3432	2.597(0.2%)	2.610(1.4%)	−0.5	LMR
^{134}Cs	336-80	23.64(1.0%)	23.13(1.0%)	+1.7	PTB
^{137}Cs	1901	2.424(1.5%)	2.450(1.0%)	−1.1	LMR
^{137}Cs	070	22.80(1.3%)	22.43(1.0%)	+1.6	IAEA
^{137}Cs	74-092	22.64(1.7%)	22.31(1.0%)	+1.5	AMR

[a]Uncertainties are given in parentheses as quoted by the vendor; dpm = disintegrations per minute.
[b]Uncertainties given in parentheses are statistical only.
[c]PTB: Physikalisch-Technische Bundesanstalt, Berlin; IAEA: International Atomic Energy Agency, Vienna; OMH: National Office of Measures, Budapest; AMR: American International Limited, Amersham, United Kingdom; LMR: Laboratoire de Metrologie des Rayonnements Ionisants, Saclay.
[d]The Ge(Li) detectors associated with systems 1 and 2 had coaxial geometry with active volumes of 46 and 57 cm^3, respectively.

decay of the ^{57}Co monitor source and was $<0.5\%$ for all measurements of ^{7}Be activity and system backgrounds.

The background in the Ge(Li) spectrometer systems was checked with a ~ 14 h count before and after each extended count of the ^{7}Be implanted detectors. Observed background rates ranged from <0.1 counts h^{-1} channel^{-1} (1) to <0.2 counts h^{-1} channel^{-1} (2) and yielded no distinguishable peak structures in the vicinity of the 477.6-keV photopeak. A composite plot showing a peak and background spectrum is displayed in Fig. 3. Finally, any ex-

TABLE III. Measured source-strength reproducibility as a function of source-detector separation for a chemically separated ^{7}Be source.

Count number	Measured source strength (dpm×10^6)[a]	Source-detector distance (cm)[b]
1	3.196	17.78
2	3.198	14.78
3	3.174	12.78
4	3.187	10.78
5	3.159	8.78
6	3.163	6.78
7	3.174	5.78
8	3.155	4.78
9	3.190	3.78
10	3.195	3.28
11	3.161	1.78
Avg	3.177±0.016(0.5%)	

[a]Statistical uncertainty for each value is $\sim 0.5\%$ (dpm = disintegrations per minute).
[b]Estimated position uncertainty ±0.01 cm.

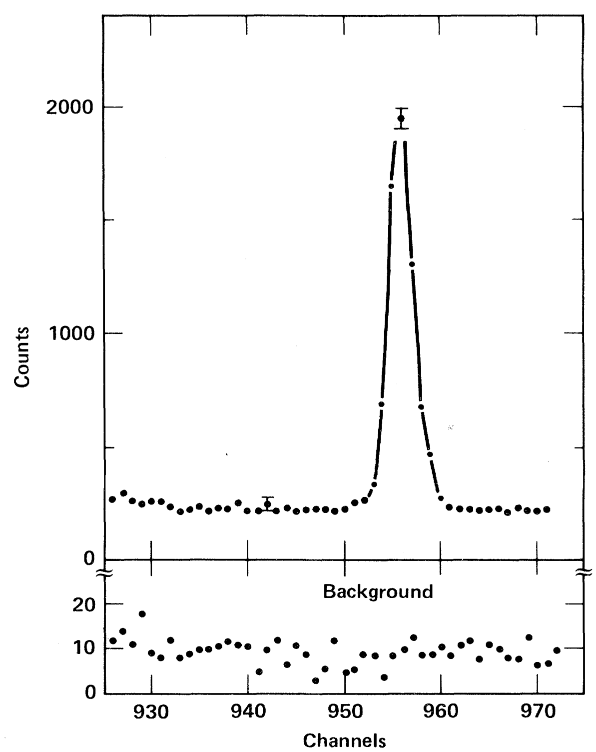

FIG. 3. A typical Ge(Li) spectrum showing the prominent 477.6 keV γ-ray line. The total counting time for this spectrum was 13.31 d at a source to detector distance of 6.35 cm. The lower portion shows the background observed during a 14 h count immediately preceding the photopeak measurement.

traneous ^{7}Be activity that might have been deposited outside the sensitive area of the detector was estimated by counting the various mechanical components which supported the silicon detector during the implantation runs. Counting times ranging from 6 to 20 h on selected components showed no unexpected ^{7}Be activity above background.

The results of the γ-ray counting measurements, after appropriate corrections for radioactive decay, were combined with the measurements of the number of ^{7}Be ions deposited on each detector to determine the electron capture branching ratio.

III. RESULTS

The results from the three separate runs and their respective γ-ray counts are summarized in Table I. Also summarized are sources of uncertainty associated with each measurement. From these results we obtain a weighted average for the branching ratio of $10.7\pm0.2\%$. This value together with other measurements[7–15,21] would give a weighted average and standard deviation of $10.45\pm0.05\%$. If the value suggested in Ref. 6 is excluded the total weighted average becomes $10.44\pm0.05\%$.

These new measurements appear systematically slightly higher than the previous measurements (which were mostly based on Auger electron spectra). This discrepancy, however, is probably not significantly greater than the inherent possible systematic uncertainties in the two techniques. We conclude that the solar neutrino problem and the discrepancies in the ^{3}He$(\alpha,\gamma)^7$Be cross section cannot be resolved by a new branching ratio for ^{7}Be.

IV. THEORETICAL APPLICATION

The present value for the ^{7}Be decay branching ratio is also of particular interest[25] in understanding Gamow-Teller (GT) transitions in nuclei. One reason is that phenomenological effective interactions exist[1] for simple p-shell nuclei like ^{7}Li and ^{7}Be, and therefore wave functions for the ground and first excited states in ^{7}Li are available. Furthermore, since the decay of ^{7}Be populates two states in ^{7}Li, some unique details of the wave functions of these states can be elucidated by the branching ratio.

The branching ratio BR for ^{7}Be decay can be written in terms of electron capture transition probabilities as follows:

$$\mathrm{BR}=\frac{\lambda_1}{\lambda_0+\lambda_1}\,, \tag{2}$$

where λ_0 (or λ_1) is the decay rate to the ground (or first excited) state. Equation (2) can be written in terms of the reduced GT and Fermi (F) matrix elements [B(GT) and B(F)] and Fermi integral functions f as

$$\mathrm{BR}=\frac{B_1(\mathrm{GT})}{B_1(\mathrm{GT})+\left[\dfrac{f_0}{f_1}\right](B_0(\mathrm{GT})+B_0(\mathrm{F}))}\,. \tag{3}$$

From the angular momentum selection rules, Fermi decay is only permitted to the ground state of ^{7}Li.

For electron capture decay the Coulomb correction to f is energy independent and cancels out. Therefore, the ratio of Fermi integral functions, f_0/f_1, can be written analytically as

$$f_0/f_1=\left[\frac{W_0+1+s}{W_1+1+s}\right]^2\cong\left[\frac{Q_0}{Q_1}\right]^2=5.031\,, \tag{4}$$

where W is the total relativistic energy available for positron decay, and Q_0 (or Q_1) is the electron-capture decay energy for the ground state (or first excited state). The approximation in Eq. (4) neglects the K-shell binding energy,

$$s=[1-(z/137)^2]^{1/2}-1=-4.3\times10^{-4}\,.$$

The reduced GT transition probabilities, B(GT), on the other hand, relate directly to certain properties of the ^{7}Li wave functions, i.e.,

$$B(\mathrm{GT})=|\langle \mathrm{CGT}\,|\,\phi_i\rangle|^2\,, \tag{5}$$

where $\langle \mathrm{CGT}|$ is the collective GT state which contains all of the GT strength resultant from the operation on the ^{7}Be ground state with the GT operator,

$$|\,\mathrm{CGT}\rangle=\vec{\sigma}\cdot\tau^+\,|\,^7\mathrm{Be\ g.s.}\rangle\,. \tag{6}$$

The states in the CGT will predominantly correspond to spin and isospin transitions of a $1p_{3/2}$ proton in ^{7}Be to a $1p_{3/2}$ or $1p_{1/2}$ neutron in ^{7}Li. Hence, the branching ratio specifically identifies the relative mixing

$$[(\pi p_{3/2})(\nu p_{3/2})^2]$$

and

$$[(\pi p_{3/2})(\nu p_{3/2}(\nu p_{1/2})]$$

configurations in the ^{7}Li ground and first excited states.

The Fermi reduced transition probability, B_0(F), should exhaust the sum rule for the $\frac{3}{2}^-\to\frac{3}{2}^-$ mirror decay, and hence, to a good approximation,

$$B_0(\mathrm{F})=(g_v/g_a)^2=0.64\,.$$

Reduced GT transition probabilities to the ground and first excited states in ^{7}Li have been determined[1,26] to be 1.62 and 1.33, respectively, from shell-model calculations based on phenomenological p-shell effective interactions. Equations (2) and (3) would then predict a branching ratio of 10.5%. This value is in excellent agreement with the average experimental ratio of 10.4%. The implication is then that the shell model wave functions correctly describe the degree of Gamow-Teller state mixing into the ^{7}Li ground and first excited states. It is of interest that these theoretical results support a value of 10.4% for the experimental ^{7}Be branching ratio since extensive modifications to the wave functions would be necessary to increase the predicted ratio to 15.4%.

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