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PHYSICAL REVIEW C VOLUME 3, NUMBER 2 FEBRUARY 1971

# Internal Bremsstrahlung Accompanying the Orbital-Electron Capture in $^7$Be

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(Received 3 Septmeber 1970)

The spectrum of the internal bremsstrahlung accompanying the electron-capture decay of $^7$Be has been measured in coincidence with the 477-keV $\gamma$ rays. From the extrapolated end-point energy of the spectrum, the transition energy to the first excited state of $^7$Li was determined to be $395 \pm 25$ keV. The intensity of the internal bremsstrahlung in the energy range from 120 to 360 keV was determined to be $(5.45 \pm 1.1) \times 10^{-5}$ per ordinary nonradiative 1*s* capture (based on a theoretically predicted *L/K* ratio of $12.4 \times 10^{-2}$) as compared with the value of $7.86 \times 10^{-5}$ predicted by the theory of Martin and Glauber.

## INTRODUCTION

When a nucleus decays by orbital-electron capture, in a certain fraction of the decay events the transformation is accompanied by electromagnetic radiation usually referred to as internal bremsstrahlung (I.B.). The low intensity of this radiation as compared with the incidence of the ordinary nonradiative capture, makes the investigation of its properties very difficult. The theory of radiative orbital-electron capture for allowed transitions was developed by Martin and Glauber.[1,2] They gave expressions for the partial spectra corresponding to capture of electrons from various atomic shells. The total intensities of the spectra were related to the ordinary nonradiative 1*s* electron capture. The approximations used in the theory are most accurate for light nuclei. The theory is fully relativistic for 1*s* electron capture. Relativistic corrections are introduced as factors multiplying the nonrelativistic expressions for the *s*-state I.B. spectra. These correction factors do not depend substantially on the energy and therefore they do not affect noticeably the shapes of the spectra. A nonrelativistic theory developed earlier by Morrison and Schiff[3] predicted essentially the same shape for the 1*s* spectrum. The intensities of the spectra, however, depend crucially on the magnitude of the relativistic corrections.

The shapes of total spectra and partial 1*s* spectra have been measured for a number of capture transitions between various nuclei and found to be in good agreement with the theory.[4,5] The intensities of both the total and 1*s* partial I.B. spectra have also been measured[6] for a few cases of allowed capture transitions. These intensities were found to be lower than the theoretically predicted ones for moderately heavy nuclei.

*Note added in proof*: After this paper was submitted for publication an article concerning the intensity of the I.B. in the decay of $^{54}$Mn by Kadar, Berenyi, and Myslek was published.[6a] Their result is in good agreement with the theory of Martin and Glauber.

Since the calculations performed by Martin and Glauber are expected to be most accurate for transformations between nuclei with small values of atomic number, it was of interest to measure the magnitude of the intensity of the I.B. for a nucleus with a very low value of $Z$.

In light nuclei it is difficult to separate the $1s$ I.B. spectrum from the other components of the total I.B. spectrum by performing a coincidence measurement with the $K$ x rays. As a consequence, it is difficult to check the predictions of the theory concerning the $1s$ spectrum only. It can, however, be expected on the basis of the theory, and has been shown experimentally, that for light nuclei the contribution of the partial spectra corresponding to capture form $nl$ states is appreciable only for $2s$ states, where it amounts to about 10% of the intensity of the total I.B. at not too low energies.

The outlined considerations have prompted us to undertake a study of the I.B. spectrum accompanying the electron-capture decay of $^7$Be, which is the lightest atom in which nuclear capture of orbital electrons takes place. Preliminary results of these measurements have been reported earlier.[7]

## MEASUREMENTS, RESULTS, AND DISCUSSION

The decay scheme[8] of $^7$Be is shown in Fig. 1. The bremsstrahlung spectrum corresponding to the electron-capture transition to the first excited state was measured in coincidence with the 477-keV $\gamma$ rays. The experimental procedures and apparatus were described earlier.[6,9] In the present measurement it was found advantageous to use a 1.8-mm lead filter between the source and the NaI(Tl) crystal from which signals were fed to the single-channel analyzer set on the 477-keV photopeak. This was done in order to absorb the backscattered $\gamma$ rays and thus practically eliminate the suppression of the upper part of the Compton distribution, which we have discussed earlier.[9] The shape of the unsuppressed distribution could be determined with better accuracy. This was important because the I.B. spectrum rested on the Compton distribution of the 477-keV line recorded due to random coincidences, and since the intensity of the I. B. was low in the region near the edge of this distribution, it was essential to determine its shape with good accuracy.

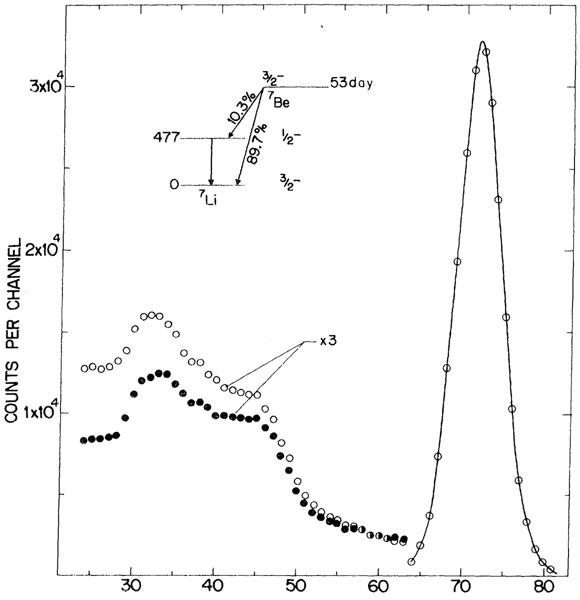

FIG. 1. The spectra obtained in coincidence with the 477-keV $\gamma$ line of $^7$Be. The true-coincidence spectrum is shown with open circles. The random-coincidence one with solid circles. In the region of the 477-keV photopeak the two spectra overlap.

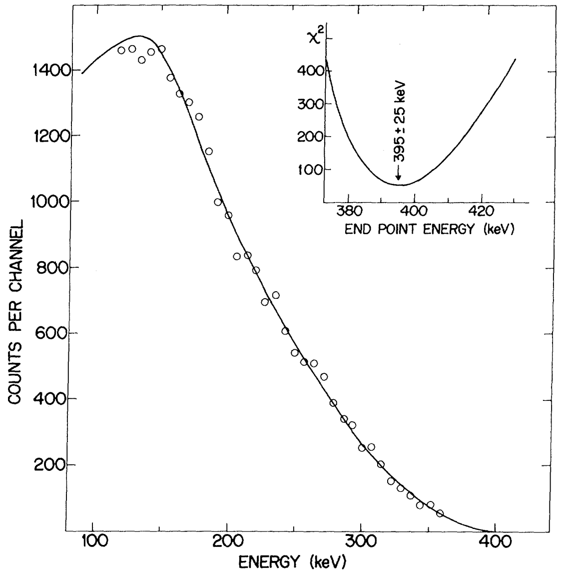

FIG. 2. The I.B. spectrum obtained as a difference between the two spectra of Fig. 1 (open circles). The solid line shows the best-fitting theoretical spectrum transformed into a pulse-height distribution, with the help of the response matrix of the spectrometer. Shown in the inset is the $\chi^2$ vs the end-point energy of the transition.

The intensity of the source during the measurements was, on the average, $10^4$ dis/sec leading to the first excited state of $^7$Li. The source was in the form of BeCl$_2$. It was checked for impurities with a Ge(Li) detector. The impurities were found to contribute less than $10^{-6}$ of the transition rate to the first excited state of $^7$Li. The random- and true-coincidence spectra were recorded simultaneously and are shown in Fig. 1. The measurements were performed for energies higher than 120 keV. The bremsstrahlung spectrum, shown

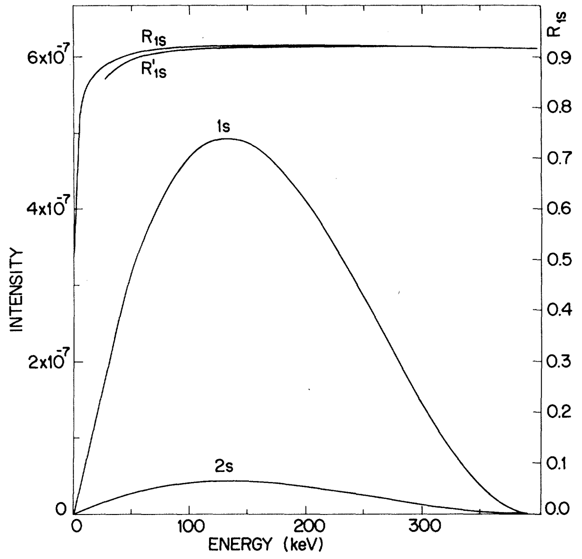

FIG. 3. Theoretical 1*s* and 2*s* I.B. spectra and the relativistic correction factor $R_{1s}$ obtained using expressions (4.3) and (4.5) of Martin and Glauber (Ref. 1).

in Fig. 2, was obtained by subtracting the random-coincidence spectrum from the true one. The low intensity of the source assured a favorable true-to-random ratio in the region where the internal bremsstrahlung appeared.

The theoretical I.B. spectra corresponding to 1*s* and 2*s* electron capture are shown in Fig. 3. The 1*s* spectrum was calculated using the relativistic correction factor $R_{1s}$ given by expressions (4.3a) and (4.3b) of Martin and Glauber.[1] This correction factor is also shown in Fig. 3. For comparison we have calculated the correction factor using expressions (4.5a) and (4.5b) of Ref. 1. The results (labeled $R'_{1s}$) are shown in Fig. 3. The agreement between the values of the correction factor obtained using the two expressions is very good in the energy range of interest.

The observed spectrum was compared with the theoretical one using a $\chi^2$ computer fitting routine as was described earlier.[9] The fitting was done by transforming the total theoretical spectrum into a pulse-height distribution using the response matrix of the detector. The end-point energy of the spectrum was used as a free parameter in the fitting procedure. The best-fitting theoretical spectrum is shown in Fig. 2. It corresponds to a transition energy of 395 ± 25 keV, which is in good agreement with the value of 384 keV given by Mattauch, Thiele, and Wapstra.[10] For the 34 points used in the fit the minimum value of the $\chi^2$ corresponds to a confidence level of ~4%.

In order to calculate the ratio of the probabilities of the radiative capture to the nonradiative 1*s* capture, one has to know the *L*/*K* capture ratio, the experimental value of which is not available. The theoretical value of this ratio extrapolated from the results of the calculations of Brysk and Rose[11] is 0.04, and from the calculations of Winter[12] is 0.0332. These values must be multiplied by the exchange-overlap correction introduced by Bahcall,[13] which, extrapolated to $z = 4$, is[14] $X^{L/K} = 3.74$. Therefore, the *L*/*K* ratio, corrected for exchange and overlap, is $14.9\times10^{-2}$ based on the theory of Brysk and Rose or $12.4\times10^{-2}$ based on the calculations of Winter. It should be noted that both the correction factor $X^{L/K}$ calculated by Bahcall and the *L*/*K* ratio calculated by Winter were obtained using the same set of Hartree-Fock wave functions given by Watson and Freeman.[15] The result of Brysk and Rose was obtained by using a different set of wave functions. The values of the *L*/*K* ratio given by Brysk and Rose corrected for exchange and overlap are generally in better agreement with the experimentally measured values in the range of atomic numbers $13 \leq z \leq 37$.[12,14] For lower atomic numbers, however, no experimental data exist to compare with the theory. The ratio $r_{\text{exp}}$ of the $1s + 2s$ radiative capture to the nonradiative 1*s* electron capture in the energy range 120–360 keV is given in Table I for all four calculated values of the *L*/*K* ratio. These values are compared with the values of $r_{\text{theor}} = 7.86\times10^{-5}$ predicted by the theory of Martin and Glauber for the same energy range, and the ratios of the observed to the predicted intensities are given in the last

TABLE I. The intensity of the I. B. in the energy range from 120 to 360 keV for the various theoretically predicted values of the *L*/*K* ratio. $r_{\text{exp}}$ and $r_{\text{theor}}$ are the experimental and the theoretical ratios of the $(1s+2s)$ radiative- to the 1*s* nonradiative-capture probabilities.

| | Brysk and Rose (Ref. 11) | | Winter (Ref. 12) | |
|---|---|---|---|---|
| | Uncorrected | Corrected | Uncorrected | Corrected |
| *L*/*K* | 0.04 | 0.149 | 0.0332 | 0.124 |
| $10^5\times r_{\text{exp}}$ | 5.05 ± 1.0 | 5.00 ± 1.0 | 5.57 ± 1.1 | 5.45 ± 1.1 |
| $r_{\text{exp}}/r_{\text{theor}}$ | 0.64 | 0.71 | 0.64 | 0.69 |

line of the table. As can be seen, the uncertainty in the theoretical value of the $L/K$ ratio does not introduce a large uncertainty in the intensity ratio, which is generally in good agreement with our previous results[6,9] for $^{54}$Mn and $^{57}$Co as well as the results of Biavati, Nassif, and Wu.[16]

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*Work supported in part by a City University Summer Research Grant.

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# Isospin Impurities in the Nuclear Ground States*

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(Received 14 September 1970)

Isospin impurities in the ground states of some even-even nuclei are calculated in the shell model including the effective residual interactions and in the three-fluid model. It is shown that the residual interactions reduce the calculated impurities by an order of magnitude. The impurities given by the three-fluid model are even smaller. Comparison of the present results is made with those of the two-fluid model.

## I. INTRODUCTION

The discovery of analog states prompted a great deal of interest in isospin mixing in nuclei. In the theoretical considerations the assumption is generally made that the nuclear ground state has pure isospin.[1] The purpose of this paper is to examine this assumption.

A number of calculations on the ground-state isospin impurities exist. Most of them[2] are in the shell-model formalism in which the isospin impurities are extracted from the Coulomb distortion of the single-particle proton orbitals with respect to neutron orbitals. The results thus obtained are of the order of a few precent – one to two orders of magnitude higher than what one would expect from the data on superallowed Fermi $\beta$ decays $(0^+ \rightarrow 0^+, \Delta T = 0)$.[3,4]

The reason for such large and experimentally incorrect values (as, for example, obtained in