Characteristics of a weakly ionized non-neutral plasma

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A scheme that allows stable confinement of a weakly ionized non-neutral plasma is discussed. The method requires the forced rotation of the neutral gas within the trap about an axis that roughly coincides with the trap's magnetic and mechanical axes. A number of the basic equilibrium and nonequilibrium properties of such a trapped plasma are calculated. © *1996 American Institute of Physics.* [S1070-664X(96)00408-9]

I. INTRODUCTION

Recent progress has allowed trapping of plasmas consisting of a number of different charged particle species[1–4] in a variety of different types of Penning traps.[1,5–7] We here broaden the scope of non-neutral plasma physics to include weakly ionized plasmas confined through the combined action of the trap's electric and magnetic fields and the forced rotational motion of a neutral gas. Non-neutral plasmas provide a useful experimental medium for the study of basic plasma physics, as well as other areas of physics such as fluid mechanics, atomic physics, nuclear physics, and particle physics. The weakly ionized non-neutral plasma may provide a similarly useful tool for the study of the physics of partially ionized plasmas and for the study of gas-phase ion chemistry.

A weakly ionized non-neutral plasma (WINP) differs less from the conventional fully ionized non-neutral plasma than one might expect. It has long been known that the inevitable presence of a small amount of (nonrotating) neutral gas leads to significant transport over long timescales.[8–11] This work merely extends this notion to the point of describing a class of non-neutral plasmas for which confinement is assured, rather than hindered, by the action of neutral gas. Although we expect that experimental realizations of a WINP will utilize atmospheric pressure gas, both higher and lower pressures are certainly possible. As discussed in Section III on transport properties, the behavior expected of a fully ionized non-neutral plasma is recovered as the trap neutral gas pressure is lowered. Systems exploiting the physical properties described in this paper are likely to exhibit three properties: the neutral species density greatly exceeds the ionic species density, the ion neutral collision frequency greatly exceeds the gyration frequency, and rotational motion of the neutral gas is deliberately imposed. We shall take these three conditions as a suitable operating definition of a WINP.

A trap designed for confinement of a weakly ionized non-neutral plasma is superficially similar to a conventional Malmberg/Penning trap.[1] Figure 1 shows the essential features of such a WINP trap. The trap has a strong axial magnetic field and a conducting outer wall divided into segments for plasma measurement and control. ''Gates'' at each axial end of the plasma must be maintained at a potential sufficient to repel the plasma and prevent particle loss. Unlike the conventional Malmberg/Penning trap, however, a neutral gas must be present and must be rotated to provide a radially inward force on the plasma and ensure confinement.

Applications for a weakly ionized non-neutral plasma would take advantage of the trap's ability to store and manipulate ionic species in the presence of neutral gas. The trap could be hooked to a mass spectrometer to allow species identification after a fixed period of time. The trap provides an environment with a well characterized Maxwellian energy distribution at a controllable temperature T (the temperature of the neutral gas). Switching between positive and negative ion confinement requires only a change in the direction of the magnetic field or gas rotation direction. Delicate and/or heavy particles such as clusters could be held and studied in a ''gentle'' environment. In addition, the trap could provide a relatively intense, low-energy source of a particular ion species for surface implantation or molecular chemistry. Finally, a long trap might allow spectroscopy of unprecedented accuracy to be performed on ionic species.

This paper describes both the properties and the confinement of a weakly ionized non-neutral plasma. Section II analyzes the equilibrium state of a weakly ionized non-neutral plasma and demonstrates that it is formally identical to that of a fully ionized plasma. Several minor differences resulting from the presence of neutral gas are discussed. Section III describes the transport that a nonequilibrium plasma must undergo. Neither energy nor angular momentum are conserved by the ion-neutral collisions that govern dynamics in a WINP. As a result, the mechanisms and consequences of nonequilibrium transport differ from their analogs in a fully ionized plasma. Section IV considers a variety of issues involved in the implementation of this trapping scheme in the laboratory.

II. EQUILIBRIUM

The equilibrium state is characterized by a balance of particle fluxes. A treatment in terms of fluxes rather than forces is more natural for a WINP because its evolution is governed by diffusion and mobility. Figure 2 contains a diagram showing the radial fluxes that must balance for an infinitely long WINP and, for comparison, the forces that must balance for a similar fully ionized non-neutral plasma. The four fluxes that must balance are the magnetic, electric, diffusive, and inertial fluxes.

A number of additional assumptions simplify calculation of the characteristics of a WINP. The plasma, trap, and magnetic field are assumed to be cylindrically symmetric and

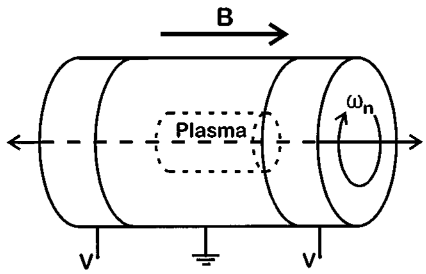

FIG. 1. Diagram showing the essential components of a weakly ionized non-neutral plasma trap. As drawn, the trap confines only positively charged ions.

aligned. The direction of the magnetic field is taken to be along the z axis so that the magnetic field can be written as $\mathbf{B}=B\hat{\mathbf{z}}$. The possible consequences of poor symmetry are discussed in Section IV A. The plasma density at the wall radius is assumed to be sufficiently low that wall loss can be neglected. A plasma in ''contact'' with the outer wall will lose charge and shrink radially in much the same way as a conventional fully ionized non-neutral plasma. The neutral gas within the trap is assumed to flow in the purely azimuthal direction with a constant angular velocity, i.e., $\mathbf{v_n}=\omega_n\hat{\boldsymbol{\theta}}$. The consequences of both turbulent and sheared neutral gas flow are also discussed in Sections IV B and IV C. Finally, the neutral gas is assumed to have a constant density and pressure. Although laboratory conditions are likely to satisfy this assumption, its violation would only lead to a dependence of the diffusion and mobility constants D and μ on radial position.

The magnetic flux results from the magnetic force that ions must feel as they are ''dragged'' azimuthally along with the neutral gas. This magnetic force is related to the Hall effect emf that arises in a wire carrying current across magnetic field lines. In the present case, current results from forced neutral gas motion, and the emf is the force that opposes the natural outward expansion of the plasma. The magnetic force has a magnitude $q(\mathbf{v_n}\times\mathbf{B})/c$, in Gaussian units, where q is the signed ionic charge. Assuming singly charged ions, this force becomes $-(e\omega_n Br/c)\hat{\mathbf{r}}$, where e is the magnitude of the electron's charge and $\hat{\mathbf{r}}$ is the radial unit vector. This force leads to a magnetic particle flux given by

$$\mathbf{\Gamma}_B=-\left(\frac{\mu n\omega_n Br}{c}\right)\hat{\mathbf{r}}, \tag{1}$$

where μ is the ion mobility and $n(r)$ is the plasma's ion density as a function of radial position. Similarly, the electric force $q\mathbf{E}(r)$ leads to an electric flux given by

$$\mathbf{\Gamma}_E=n\mu\mathbf{E}, \tag{2}$$

where E is the electric field arising from both the plasma itself and from the trap's confining electric field. The diffusive flux is given by Fick's law,

$$\mathbf{\Gamma}_D=-D\nabla n, \tag{3}$$

where D is the ionic diffusion constant. This diffusion constant is calculated for each particular gas composition and pressure. The effect of various types of ion-neutral interactions including charge-exchange reactions must be accurately accounted for.[12] Finally, the inertial force on an ion of mass m_i within a gas containing a single species of mass m_n has the value $(m_i-m_n)\omega_n^2 r\hat{\mathbf{r}}$, resulting in an inertial ion flux of

$$\mathbf{\Gamma}_M=\delta m n\mu\omega_n^2 r/e\hat{\mathbf{r}}, \tag{4}$$

where $\delta m=(m_i-m_n)$.

Equation (4) implies that the inertial flux can be either inwardly or outwardly directed depending on the sign of δm. For the case that $m_i<m_n$, the inertial flux acts to reinforce the magnetic flux, and there can be *no* Brillouin limit[13] to the density that can be achieved by arbitrarily increasing the frequency of neutral gas rotation. In practice, however, such rotation frequencies are unlikely to be achieved. The inertial force should be negligible whenever $\Gamma_M/\Gamma_B\ll 1$, i.e., $(\delta m/m_n)(\omega_n/\Omega)\ll 1$, where $\Omega=eB/m_i c$ is the ion cyclotron frequency. Since experimental limitations on the maximum possible gas rotation speed will ensure that $\omega_n/\Omega\ll 1$, we hereafter neglect inertial terms under the assumption that the plasma is composed of relatively light ions.

The cold plasma limit is obtained by setting $T=D=0$, where T is necessarily the temperature of both the neutral gas and the plasma. Without a diffusive flux, the cold plasma equilibrium profile can be calculated from $\mathbf{\Gamma}_B+\mathbf{\Gamma}_E=0$. For the case of an infinitely long plasma and no contribution from the trap's confinement fields, the electric field can be expressed as $E(r)=(4\pi e/r)\int_0^r r'n(r')dr'$. The condition for equilibrium therefore reduces to

$$n(r)=n_0=\frac{\omega_n B}{2\pi ec}, \tag{5}$$

where n_0 is the constant ion density throughout the plasma. Equation (5) is similar to the relation that also describes the $T=0$ equilibrium profile for a conventional fully ionized non-neutral plasma.[14] The difference between the two cases centers around the fact that the rotational frequency of a WINP is set by the externally imposed neutral gas flow, whereas the rotational frequency of a fully ionized plasma is determined purely by $\mathbf{E}\times\mathbf{B}$ drift. Although a WINP at equi-

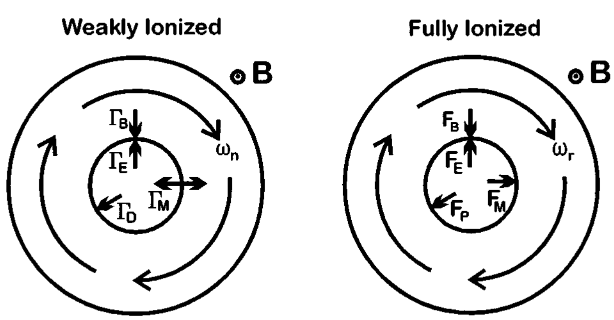

FIG. 2. Diagram showing the radial fluxes and forces that must balance at equilibrium for fully and weakly ionized non-neutral plasmas, respectively. The fluxes in a WINP are magnetic (Γ_B), electric (Γ_E), diffusive (Γ_D), and inertial (Γ_M). The forces within a fully ionized non-neutral plasma are magnetic (F_M), electric (F_E), pressure (F_P), and inertial (F_M). The rotational frequencies for the two cases are ω_n and ω_r, respectively. The indicated rotational and magnetic directions are valid for positively charged ions.

librium rotates at a speed equal to that expected for an identical plasma in vacuum undergoing $\mathbf{E}\times\mathbf{B}$ drift, this distinction remains crucial. Drifts generally do *not* occur when $\nu_{in} \gg \Omega$, where ν_{in} is the ion-neutral collision frequency.[15] (An ion in crossed $\mathbf{E}$ and $\mathbf{B}$ fields moves in two completely different directions depending on whether $\nu_{in} \gg \Omega$ or $\nu_{in} \ll \Omega$.)

The profile of a WINP with $T>0$ is best found by comparison with the $T>0$ equilibrium profile of a fully ionized non-neutral plasma. We show below that the equations governing the detailed equilibrium profile for the two cases are isomorphic. Finite temperature results in the addition of diffusion for a WINP and the addition of pressure forces for a fully ionized plasma. A fully ionized non-neutral plasma must satisfy $\mathbf{F}_B+\mathbf{F}_E+\mathbf{F}_P=0$, where

$$\mathbf{F}_B=-(e\omega_r Br/c)\hat{\mathbf{r}}, \tag{6}$$

$$\mathbf{F}_E=e\mathbf{E}, \tag{7}$$

$$\mathbf{F}_P=-(T\nabla n)/n \tag{8}$$

are the magnetic, electric, and pressure gradient forces, respectively. Equations (6)–(8) are formally equivalent to Eqs. (1)–(3) provided that the Einstein relation $\mu=eD/T$ is used to relate mobility and diffusion and provided that the fully ionized plasma rotation frequency ω_r is identified with the neutral gas rotation frequency ω_n.

The precise equivalence described above implies that the possible equilibrium shapes and profiles of weakly ionized non-neutral plasmas must be *identical* to those of fully ionized plasmas. It is therefore possible to apply much of the previous work on plasma shapes and profiles to a WINP.[16–18] Specifically, the plasma must have an interior of roughly constant charge density and a relatively thin "edge" with a thickness of roughly the Debye length $\lambda_D=\sqrt{T/4\pi ne^2}$. As with fully ionized non-neutral plasmas, wall loss of ions should be negligible because the exterior plasma density decreases approximately exponentially with radius.

III. TRANSPORT

Ion transport within a WINP is constrained by none of the usual angular momentum or energy conservation principles affecting fully ionized non-neutral plasmas.[19,20] The plasma can gain or lose both energy and angular momentum via interaction with the rotating neutral gas. At atmospheric pressure, the angular momentum associated with the rotational motion of the neutral gas greatly exceeds the canonical angular momentum of the plasma's equilibrium state. In any case, the torque necessary to cause transport within the WINP must be externally supplied as the force necessary to maintain the neutral gas motion. Initially, the plasma's density and radius may be far from their required equilibrium values. Rapid initial transport causes the plasma to shrink or expand radially, reaching a state of approximate equilibrium.

The approximate timescale for this initial transport can be found by setting $T=D=0$ and assuming an infinitely long plasma of constant density $n_0+\delta n$, where n_0 is the cold-plasma equilibrium density given by Eq. (5). Charge conservation requires that

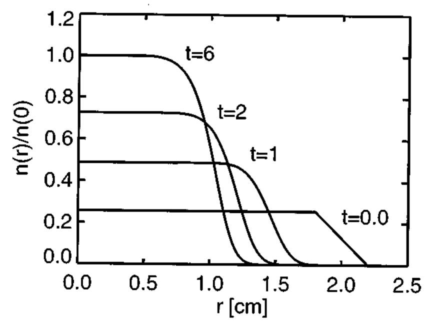

FIG. 3. Numerically calculated radial density profile at four different times in the evolution of a WINP toward equilibrium. The indicated times are measured as multiples of the "fast" transport time constant, $\tau_f=c/2\mu\omega_n B$. The initial conditions for this plasma were chosen such that the equilibrium state would have $r_p/\lambda_D=10$.

$$\frac{\partial(n_0+\delta n)}{\partial t}=\nabla\cdot(\mathbf{\Gamma}_B+\mathbf{\Gamma}_E). \tag{9}$$

Elimination of n_0 yields a differential equation that can be solved for the approximate time dependence of a small density perturbation $\delta n(t)\approx\delta n(0)e^{-t/\tau_f}$, where the "fast" transport time constant is given by

$$\tau_f=c/\mu\omega_n B. \tag{10}$$

As the neutral gas pressure is lowered, one expects to recover the behavior of a conventional, fully ionized non-neutral plasma. The transition occurs at neutral gas pressures such that $\nu_{in}\approx\Omega$. Above this pressure, the plasma is effectively forced to follow the neutral gas flow and transport occurs as described in this section. Below this pressure, the plasma's rotational velocity is effectively set by the action of $\mathbf{E}\times\mathbf{B}$ drift. Although interaction with the neutral gas may still lead to significant transport, the relations described herein will cease to be valid.

Before a WINP can completely reach equilibrium, slower diffusive transport must occur primarily to establish the plasma's detailed edge profile. Since the expected size of the region over which diffusion should be important is roughly λ_D, the slow diffusive timescale can be written

$$\tau_s=\lambda_D^2/D. \tag{11}$$

Note that the ratio of the slow diffusion timescale to the fast electrostatic timescale is $\tau_s/\tau_f=1/2$. Figure 3 shows evolution of the radial profile of a plasma with initial density and temperature such that $r_p/\lambda_D=10$ at equilibrium, where r_p is the plasma radius. The data displayed in Figure 3 were generated by a straightforward numerical solution to the relation $\partial n(r)/\partial t=\nabla\cdot(\Gamma_D+\Gamma_E+\Gamma_B)$, which describes conservation of charge for this dynamical system. As expected, equilibration is nearly complete after a time of $6\tau_f$. Interestingly, the tendency for ion-neutral collisions to lead to the standard non-neutral plasma equilibrium profile has also been noted by recent work in conventional fully ionized plasmas.[10,11]

Confinement of a WINP in a trap with idealized neutral gas flow should be extraordinarily stable. Other than the exponentially small loss of ions at the outer wall and axial ends, there is no transport process by which the plasma can be lost. *Any* deviation of the plasma's profile from the equilibrium profile will be undone on roughly the timescale τ_f.

IV. IMPLEMENTATION

The conditions necessary for successful trapping of a weakly ionized non-neutral plasma should be obtainable in practice. A sharp point (or array of points) undergoing stable corona discharge could supply at least 10^{-3} A of either positive or negative ion current.[21,22] Such a plasma source would function at atmospheric pressure in a wide variety of chemical environments. A rapidly rotating, turbulence-free neutral gas could be provided by a technology similar to that used for sample spinning in NMR chemical analysis, or any of a variety of centrifugal separation technologies. Rotation rates of 1–10 kHz should be achievable for a small laboratory trap.

Numerical evaluation of typical plasma characteristics requires an estimate of the ion mobility and diffusion constants within the trap. As an example, we assume a plasma consisting of singly charged, lightweight ions in air at a temperature of 0.025 eV. The ion-neutral collision frequency for a variety of nonpolar neutral species is roughly $\nu_{in} \approx (10^{-9}\ \text{cm}^3/\text{s}) n_n$, where n_n is the neutral species density. Estimation of μ and D at atmospheric pressure yields $\mu = e/m\nu_{in} = 380$ esu-s/g and $D = T/m\nu_{in} = 0.032\ \text{cm}^2/\text{s}$, where we have assumed for simplicity that the ion mass is roughly equal to the mass of the nitrogen molecule. (Identification of a particular ion species and discussion of its gas-phase chemical processes is beyond the scope of this paper.) Further assuming a trap with $\omega_n = 3000\ \text{s}^{-1}$ and $B = 10^4$ G, Eqs. (5) and (10) predict a plasma with a density $n = 3.3 \times 10^5\ \text{cm}^{-3}$ and equilibration time constant $\tau_f = 1.3$ s. The trapped plasma should exhibit collective behavior because the plasma's radius and Debye length can easily be made to satisfy $r_p/\lambda_D \gg 1$. It is also evident that the plasma is not highly correlated because the "plasma parameter" $\Lambda = n\lambda_D^3$ is much greater than one. The fact that $\nu_{in} \gg \Omega$ ensures that ion-neutral collisions are of dominant importance in the plasma dynamics. The pressure at which the transition to conventional fully ionized non-neutral plasma behavior is expected is roughly 100 mTorr.

It is interesting to note that this trapping scheme might be used to create a non-neutral plasma within an alternate medium with free, mobile charges such as a liquid or semiconductor. Since charge mobility in liquids is roughly three orders of magnitude lower than in gasses, equilibration would take many minutes. However, the roughly three orders of magnitude higher mobility in semiconductors[23] would lead to nearly complete equilibration in roughly 10^{-2} s. In a semiconductor it would be possible to create an unneutralized plasma where the more abundant species consists of trapped vacancies ("holes").

The remainder of this section describes the consequences of violating three of the primary assumptions used in the calculation of a WINP's equilibrium and nonequilibrium properties.

A. Asymmetry

The effects of mechanical, electrical, or magnetic asymmetry on the confinement of a WINP are far more straightforward and limited than on the confinement of a conventional fully ionized non-neutral plasma. For a fixed total amount of charge in a specific trap there is a unique equilibrium state. Asymmetries may modify this equilibrium state, but cannot cause a gradual evolution as occurs for fully ionized plasmas.

The general response of a WINP to asymmetries can be illustrated by considering the application of a magnetic "tilt" $B_x\hat{\mathbf{x}}$ to an otherwise symmetric trap with an axial magnetic field $B\hat{\mathbf{z}}$. Under such conditions, a fully ionized plasma remains aligned with the total magnetic field $B_x\hat{\mathbf{x}} + B\hat{\mathbf{z}}$ and experiences enhanced radial transport. In contrast, a WINP remains centered about the axis of neutral gas rotational flow, which normally coincides with the trap axis. Ions are subjected to an oscillatory force $(q\omega_n r B_x/c)\hat{\mathbf{z}}\sin(\omega_n t)$, which causes them to undergo an axial oscillation over a distance of $2\mu B_x r/c$. Even for tilt angles as large as $\pi/4$, this oscillation distance is small compared to the trap dimensions! We conclude, therefore, that poor uniformity and alignment of the trap magnetic field have no appreciable effect on WINP confinement other than a slight modification of the equilibrium state.

B. Sheared flow

An important nonideality that may be present in laboratory experiments is neutral gas flow with a rotational frequency that depends on radius. Note that under these conditions the plasma reaches a state with significant shear. We here generalize Eq. (5) to account for this possibility in a plasma that can be approximated as cold ($T \approx 0$, $D \approx 0$.) Letting $\omega_n = \omega_n(r)$ and requiring that $\Gamma_E = -\Gamma_B$ yields the relation

$$\frac{\omega_n(r) B r^2}{4\pi e c} = \int_0^r r' n(r') dr', \tag{12}$$

where the substitution $E(r) = (4\pi e/r)\int_0^r r' n(r') dr'$ has been made for the electric field. Equation (12) can be solved for the generalized cold plasma density profile

$$n(r) = \frac{(2\omega_n(r) + r d\omega_n(r)/dr) B}{4\pi e c}. \tag{13}$$

Equation (13) predicts that whenever the rotational frequency decreases faster than $1/r^2$, plasma composed of oppositely charged species are present. Equation (13) would thus be unphysical unless ions of both sign of charge are available in sufficient quantity. Ion recombination could prevent the formation of a steady-state plasma with rotation profiles of this type. It should be noted that we do not use the term "equilibrium" to describe the steady-state condition of a permanently sheared plasma because continuing interaction between the neutral gas and the external world is necessary to maintain shear in any real system.

C. Irregular gas flow

As discussed in Section A above, mechanical asymmetry should have a negligible effect on the plasma unless it disrupts the neutral gas flow. Any disruption of the neutral gas flow that leads to an irregular or turbulent flow can significantly alter the equilibrium profile or lead to loss of confinement. One approximate treatment of such flow assumes an enhanced diffusion and therefore an elevated value for D. Thus, an irregular flow increases the effective Debye length λ_D of the plasma's equilibrium profile. Should severe turbulence occur, the plasma comes into contact with the outer wall and will eventually decrease in size or be lost entirely.

D. Oppositely signed charge

The sign of charge that can be confined in a trap is given by $-\hat{\omega}\cdot\mathbf{B}/|B|$. As in a fully ionized non-neutral plasma, oppositely charged ions are attracted by the confinement potentials at the axial ends of the plasma. Since the axial electric fields within the plasma are necessarily zero, axial loss is a diffusion limited process and is therefore very slow in a long plasma. Radial transport also provides a significant loss mechanism for oppositely charged ions. The total radial flux for an oppositely charged "test ion" in an equilibrium plasma is

$$\Sigma\Gamma = \Gamma_D + \Gamma_B + \Gamma_E \tag{14}$$

$$= \Gamma_D - \Gamma_B^0 - \Gamma_E^0 \tag{15}$$

$$= \Gamma_D + \Gamma_D^0, \tag{16}$$

where Γ_D^0, Γ_B^0, and Γ_D^0 are the fluxes for the trapped species. Clearly, the test ion must eventually be expelled by the action of the purely diffusive terms in Eq. (14).

V. CONCLUSION

We have described a new type of non-neutral plasma that is weakly ionized and therefore must be confined by the forced rotation of neutral gas. A WINP is best described in terms of its similarities to and differences from conventional non-neutral plasma. Its three-dimensional equilibrium density profile is normally very similar to that of the corresponding fully ionized plasma. Although inertial effects are quite different in a WINP, laboratory plasmas are likely to correspond to fully ionized plasmas that are well below the Brillouin limit. A WINP can exist stably in a sheared state, although under these conditions energy must be externally input via the neutral gas. The transport mechanisms that drive a WINP toward equilibrium are, of course, drastically different from those in a fully ionized plasma. The plasma undergoes rapid transport leading to a state in which its density profile is appropriate for the imposed rotational frequency. Notably, it should be possible to confine a WINP indefinitely even in a trap with poor magnetic alignment or uniformity. Mechanical precision is necessary only to the extent that mechanical asymmetries can disrupt the rotational flow of the neutral gas.

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