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A SOLAR MODEL WITH LOW NEUTRINO EMISSION*

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ABSTRACT

A model for the Sun giving a reaction rate on ^{37}Cl as low as 0.5 SNU is discussed. The low neutrino emission comes about because the Sun is taken to have a core containing 0.3–0.5 $M_{\odot}$ with an unusually high concentration of iron-group metals and a low initial concentration of helium. The high opacity caused by the metals makes the core convective, a suggestion which has been made previously by other authors on an ad hoc basis. In such a situation the low neutrino emission arises for the following two reasons: (1) a high hydrogen concentration, $X \approx 0.7$, is maintained at the center; (2) convective mixing of ^{7}Be on a time scale less than the ^{7}Be destruction time minimizes the importance of $^7\text{Be}(p, \gamma)^8\text{B}$.

Subject headings: cosmology — interiors, solar — neutrinos

I. RELATION OF THE MODEL TO COSMOLOGY

The model has a core with a different composition from the exterior, containing 30 to 50 percent of the solar mass. The exterior is taken to have a composition

$$X_e = 0.70\,, \quad Y_e = 0.285\,, \quad Z_e = 0.015\,, \tag{1}$$

with Z_e made up of elements heavier than helium in the usual way. Results are not sensitive to this precise choice for the exterior—X_e could be increased to 0.755 and Y_e decreased to 0.23, for example, without the results being significantly affected.

The core is taken to be a mixture of two compositions, both of cosmological origin, one corresponding to high baryon density and the other to low baryon density. In the notation of Wagoner, Fowler, and Hoyle (1967) (WFH) these are cases of high and low h, respectively. The latter gives a composition of the kind shown in the left-hand columns of Table 3A of WFH, while the former is an extrapolation to the right-hand side of Table 3B. For $h \approx 10^4$ the reaction 3 $^4\text{He} \rightarrow {}^{12}\text{C}$, followed by subsequent α-particle additions, produces what is essentially a mixture of H and of elements heavier than Mg. Explicitly we take

$$X = 0.96\,, \quad Y = 0.04\,, \quad \mathbf{Z} = 0 \qquad (\text{low-}h\text{ case})\,;$$
$$X = 0.50\,, \quad Y = 0.00\,, \quad \mathbf{Z} = 0.50 \qquad (\text{high-}h\text{ case})\,.^1 \tag{2}$$

It is from a mixture of these compositions that the core of the Sun is supposed to have initially formed.

The motivation for considering these particular disparate cases comes from a recent paper (Hoyle 1975), in which the baryon density at the epoch of cosmological nucleosynthesis was found to be more spatially variable than is usually supposed. Previously existing stars did not evaporate and become uniformly distributed. The material of such stars provides the high-h case, while the material lying between the stars provides the low-h case. (The reader who feels this motivation to be rather remote can just think in terms of the investigation of a solar model of somewhat peculiar composition. The failure to detect ^{37}Cl absorption at a significance level of 1 SNU—compared with 5.6 SNU for a "standard" solar model—has prompted a search for low neutrino emission models, however esoteric.)

Like all stars forming shortly after the epoch of cosmological nucleosynthesis, the core of the Sun would have had very low luminosity, with little He production by the p-p chain. This was because of the high opacity of the $\mathbf{Z}$ material, the situation being like the "opacity discrepancy" found by Eddington when he attempted to obtain normal luminosities for stars of high $\mathbf{Z}$. Because of their low luminous output, such stars could not readily be seen. If present in large numbers in some galaxies, they would give high mass-to-light ratios for those galaxies. If present in large numbers on an intergalactic scale, they could provide sufficient "hidden mass" to be of cosmological importance.

The exterior of the Sun, comprising 50 percent or more of the present solar mass, is taken to have been added 4.7×10^9 years ago. To provide pressure support against gravity for the exterior, the temperature at the core surface had to be $\sim 10^7$ K. At this temperature the radiative flux in the exterior was much greater than the radiative flux in the core, because the opacity in the exterior was much less than in the core. Hence to ensure continuity of energy flow from the core to the exterior the nuclear energy generation in the core had to increase markedly, and in order to transmit this energy to the exterior the core had to become convective.

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[1] Boldface type is used for $\mathbf{Z}$ as a reminder that the high-mass-number elements are heavier than Mg, the usual C, N, O, and Ne being removed by α additions. Hence there will be no ^{13}N or ^{15}O neutrinos from the core of the Sun.

The model evolves with time through $4H \rightarrow {}^4He$ by the pp chain, which leads to an increase of the Y value within the core, from a value ~ 0.04 some 4.7×10^9 years ago to ~ 0.15 at the present day. Such an increase is rather minor in its effect on the main solar structure, however, and does not lead to any very appreciable change in the luminosity. Hence $L_\odot$ must have been close to its present-day value of 3.83×10^{33} ergs s^{-1} over the whole history of the Earth—an expectation in better agreement with geological requirements than is the standard model, which requires the Sun to have been significantly less luminous during the Earth's early history.

With the model now defined, the aim is to calculate the present-day solar neutrino emission for mixtures of the two compositions given in equation (2). Explicitly, the following three possibilities for the *present-day* composition of the core will be considered:

$$X = 0.70\,, \quad Y = 0.15\,, \quad Z = 0.15\,; \tag{3}$$

$$X = 0.75\,, \quad Y = 0.15\,, \quad Z = 0.10\,; \tag{4}$$

$$X = 0.775\,, \quad Y = 0.15\,, \quad Z = 0.075\,. \tag{5}$$

The Y values here are taken the same, because the Y value is mainly determined by $4H \rightarrow {}^4He$ over the past 4.7×10^9 years, and with $L_\odot$ constrained in our calculations to be always 3.83×10^{33} ergs s^{-1} the helium production is the same.[2]

For the core masses under consideration, 0.3–0.5 $M_\odot$, the temperature gradient at the beginning of the exterior must be markedly subadiabatic. Integrations for the exterior show that the effective polytropic index must take an initial value in the range 2.6 to 3.1, not the value 1.5 applicable in the core. This implies that the exterior, at its boundary with the core, is very stable against convection. It does not seem unreasonable, therefore, to postulate that the core and the exterior have persisted as separated material over a time scale of 4.7×10^9 years.

II. CONVECTIVE MOTIONS IN THE CORE

The velocity v of moving elements of material and the convective energy flux H per unit area which they carry are given in terms of the scale height l by the equations

$$v^2 \approx \frac{g}{T} \cdot \Delta\nabla T \cdot l^2\,, \tag{6}$$

$$H \approx \tfrac{1}{4} C_p \rho \left(\frac{g}{T}\right)^{1/2} (\Delta\nabla T)^{3/2} l^2\,, \tag{7}$$

the notation being that of Schwarzschild (1958). In a strongly convective situation the scale height becomes of the same order as the dimension of the whole convective instability, in the present case the core radius r_c—it is only in weakly convective situations, where the instability happens to be comparable with other fluctuations, that moving elements stop in mid-flight, dissolving and giving place to new elements. In the present problem nuclear energy is mainly generated very close to the center by the reaction ${}^3He({}^3He, 2p){}^4He$. Because of this central condensation of the energy supply a powerful flux, of the order of 10^{13} ergs cm^{-2} s^{-1} must arise, so large indeed that we may expect the flow to proceed outward in the most efficient manner possible, i.e., with $l \approx r_c$. It can then be shown that

$$v \approx \left(\frac{L_\odot r_c}{M_c}\right)^{1/3}\,, \tag{8}$$

the steps in establishing this result being: (i) put $H \approx L_\odot / 4\pi r_c^2$, (ii) eliminate $\Delta\nabla T$ between equations (6) and (7), (iii) put $C_p = \frac{5}{2}\Re$, $g \approx \Re T/\mu r_c$, $M_c \approx (4/3)\pi r_c^3 \rho$.

The same result can be derived more immediately from basic thermodynamics. The total energy carried from the central temperature T_0 to the core boundary at temperature T_c over the convective circulation time $t \approx r_c/v$ is $tL_\odot$. A heat engine can deliver this total partly as heat and partly as mechanical work with efficiency $1 - T_c/T_0$. For each circulation period we therefore can have

$$\text{Dynamical energy produced} \approx \tfrac{1}{2} M_c v^2 \approx (1 - T_c/T) L_\odot t\,. \tag{9}$$

Since $t \approx r_c/v$, this gives

$$v \approx \left[2\left(1 - \frac{T_c}{T_0}\right) \frac{L_\odot r_c}{M_c}\right]^{1/3}. \tag{10}$$

For M_c in the range 0.3–0.5 $M_\odot$ the core boundary temperature is about 0.6 T_0. Hence equation (10) agrees closely with (8).

[2] Taking $L_\odot = 3.83 \times 10^{33}$ ergs s^{-1} over 4.7×10^9 years, 9×10^{31} g of H have been converted to He. For a core with mass 0.4 $M_\odot$, the increment in Y has been 0.11. The value 0.15 in eqs. (3), (4), and (5) thus implies $Y \approx 0.04$ for the core material at its origin.

Putting $r_c \approx 2 \times 10^{10}$ cm, $M_c \approx 10^{33}$ g, in equation (8) gives $v \approx 4 \times 10^3$ cm s^{-1}, and $t \approx r_c/v \approx 5 \times 10^6$ s. This result is important in the neutrino problem, because it requires ^{7}Be, produced near the core center, to become distributed throughout the core. Indeed, the circulation time t is so short that all nuclear species can be taken to approximate to uniformity in their distribution throughout the core. A situation of this kind was discussed by Iben (1969). Convective models with slower circulation times have been considered by Ezer and Cameron (1968) and by Shaviv and Salpeter (1971).

III. NEUTRINO RATES

The core satisfies a polytrope of index 1.5, with the radius Z_c in Emden units related to r_c by

$$Z_c = \left(\frac{5\Re T_0}{8\pi\mu G\rho_0}\right)^{-1/2} r_c\,, \tag{11}$$

where ρ_0 is the central density and μ is the molecular weight of the core material. The calculation of neutrino rates is made subject to the constraint that nuclear energy is generated at 3.83×10^{33} ergs s^{-1} by the p-p chain. With this constraint applied, and with the core mass M_c specified, the calculation depends only weakly on the exterior, through the effect of the exterior on the determination of Z_c. Assuming a constant radiative flux in the exterior, the determination of Z_c is slightly affected by the dependence of the opacity κ on the density ρ and the temperature T. The tabular values for the opacity given by Watson (1969–70) and by Cox and Stewart (1969–70) can adequately be represented for the composition (1) by the equation

$$\kappa = 2.2\left[\frac{10^5\rho}{T_6^{\,7}}\right]^{1/3} \text{cm}^2\ \text{g}^{-1}\,, \tag{12}$$

with ρ in g cm^{-3} and T_6 in units of 10^6 K. The required value of Z_c depends rather weakly on the functional relation of ρ to T in equation (12), namely, $\kappa \propto (\rho/T^7)^{1/3}$, and not at all on the constant of this proportionality.

Calculations have been made explicitly for the case $M_c = 0.4\ M_\odot$. Neglecting a small effect due to electron screening, results expressed in terms of the μ, X values of the core material are:

$$\text{Central temperature (unit } 10^6\text{K)}: T_0 = 16.044\left(\frac{\mu^{1.5}}{X}\right)^{0.2606}; \tag{13}$$

$$\text{Central density}: \rho_0 = 23.43X^{-2}\left(\frac{\mu^{1.5}}{X}\right)^{-1.2182} \text{g cm}^{-3}; \tag{14}$$

$$\text{Emden unit}, \left(\frac{5\Re T_0}{8\pi\mu G\rho_0}\right)^{1/2}: 1.30 \times 10^{10}\mu^{0.6091}X^{0.2606}\ \text{cm}; \tag{15}$$

$$\text{Emission rate from } ^7\text{Be}\ (e, \nu)^7\ \text{Li}: 3.95 \times 10^{36}X^{-1}\left(\frac{\mu^{1.5}}{X}\right)^{1.8015} \text{s}^{-1}; \tag{16}$$

$$\text{Emission rate from } ^8\text{B}: 7.57 \times 10^{33}\frac{1}{1+X}\left(\frac{\mu^{1.5}}{X}\right)^{5.5} \text{s}^{-1}. \tag{17}$$

The index values in these relations are determined by constants appearing in the reaction rates for ^{1}H(p, $e\bar{\nu}$)^{2}H, ^{3}He(^{3}He, 2p)^{4}He, ^{4}He(^{3}He, γ)^{7}Be, ^{7}Be(e, $\nu\gamma$)^{7}Li, ^{7}Be(p, γ)^{8}B. The values given recently by Fowler, Caughlan, and Zimmerman (1974) were used.

Fluxes at the Earth, and ^{37}Cl counting rates expressed in SNU, are given in tables 1 and 2 for the core compositions (3), (4), (5).

The contribution of pep neutrinos must be added to the rates given in these tables. In the standard model, pep neutrinos add about 0.3 SNU. In the present model, on the other hand, the contribution is only ~0.15 SNU, since

TABLE 1

FLUXES AND COUNTING RATES FOR ^{7}Be NEUTRINOS

μ	X	Flux (cm^{-2} s^{-1})	Counting Rate (SNU)
0.62992........	0.70	1.09×10^9	0.32
0.60150........	0.75	7.97×10^8	0.23
0.58823........	0.775	6.84×10^8	0.20

NOTE.—Core mass 0.4 $M_\odot$.

TABLE 2

FLUXES AND COUNTING RATES FOR ^{8}B NEUTRINOS

μ	X	Flux (cm^{-2} s^{-1})	Counting Rate (SNU)
0.62992........	0.70	2.49×10^5	0.34
0.60150........	0.75	1.13×10^5	0.15
0.58823........	0.775	7.73×10^4	0.10

NOTE.—Core mass 0.4 $M_\odot$.

the *pep* rate involves a density factor and the central density ρ_0 is low in the present model, $\sim$75 g cm^{-3} compared with $\sim$150 g cm^{-3} for the standard model. There are no ^{13}N or ^{15}O neutrinos.

The total ^{37}Cl count is thus $\sim$0.81 SNU for composition (3), is $\sim$0.53 SNU for composition (4), and is $\sim$0.45 SNU for composition (5). The count from ^{8}B neutrinos is sensitive to the composition, being small for $X > \sim 0.75$. However, once the ^{8}B neutrinos are suppressed by thus increasing X, the count changes only slowly with X. A rate somewhat less than 0.5 SNU can be obtained, but not a rate much less than 0.5 SNU. This same point has been emphasized by Ulrich (1974).

IV. THE EXTERIOR

Although the properties of the exterior do not much affect the calculations of the preceding section, the exterior imposes an interesting condition in its own right. For a radiative flux F we have

$$\tfrac{4}{3}acT^3\frac{dT}{dr} = -\kappa\rho\cdot\frac{F}{4\pi r^2}\,. \tag{18}$$

From equation (18), together with

$$\frac{dP}{dr} = \frac{\Re}{\mu_e}\frac{d}{dr}(\rho T) = -\frac{GM}{r^2}\rho\,, \tag{19}$$

the effective polytropic index n defined by

$$n + 1 = \frac{d\ln P}{d\ln T} \tag{20}$$

is given by

$$n + 1 = \frac{16\pi}{3}ac\frac{\mu_e GMT^3}{\Re\kappa F\rho}\,. \tag{21}$$

In order that an exterior can be fitted with T, P continuous at the core boundary it is necessary, with M_c specified, that an appropriate value be given to n at the beginning of the exterior.[3] Thus, with M_c stated, the left-hand side of equation (21) is determined, as also is the value of Z_c associated with the core. With Z_c known, and with ρ_0, T_0, given by the interior calculation, ρ and T on the right-hand side of equation (21) are known at the beginning of the exterior.[4] Then with $M = M_c$, and with κ given by (12), equation (21) determines the radiative flux F. Should the result be $L_\odot = 3.83 \times 10^{33}$ ergs s^{-1}?

In § II it was seen that thermodynamic efficiency in the core can be such that in each circulation time t a heat engine converts $\sim 0.4\, L_\odot t$ into mechanical work, which appears in the motion of the core material. Now mechanical energy does not continue to accumulate in the core material from one circulation period to another. The mechanical energy produced by the heat engine is transmitted to the exterior, either through dissipation into heat or by continuing into the exterior in mechanical form—for example, as outward moving sound waves. If conversion into heat takes place, the above calculation should give $F = L_\odot$; but if no conversion into heat occurs, the calculation should give $F \approx 0.6\, L_\odot$.

The calculation turns out to depend on M_c and on the composition of the core. For $M_c \approx 0.3\, M_\odot$, and for composition (3), $F \approx L_\odot$. The model in this case would have a neutrino count of $\sim$1 SNU. For $M_c \approx 0.4\, M_\odot$, and for compositions (4) and (5), the calculation gives $F \approx 0.6\, L_\odot$. Since it is the latter cases which give a neutrino count of $\sim$0.5 SNU, it follows that to obtain a consistent model with a very low count it is necessary for some 40 percent of the solar luminosity to be transmitted mechanically from the core into the exterior.

In such a model the mechanical energy must eventually be dissipated into heat. Where? When, using $F \approx 0.6\, L_\odot$ for the models in question, equations (18) and (19) are followed outward from the core boundary, the effective polytropic index is found to rise from its appropriate starting value to a value close to 4, where it is maintained until the subsurface convection zone of the Sun is reached. Without much affecting the structure (i.e., without much changing either Z_c or the starting value of n), the mechanical energy can be dissipated once the polytropic index rises close to 4. Although not necessarily requiring dissipation to occur immediately below the solar surface, this condition does require dissipation to take place outside some 80 percent of the solar mass—that is to say, in the outer half of the solar radius.

The mechanical motion in the core cannot have strict spherical symmetry, and mechanical work transmitted into the exterior would not have strict spherical symmetry. The dissipation of the work would therefore produce a situation in which the temperature distribution over gravitational equipotential surfaces was nonuniform, with the effect that large-scale circulating currents would be driven in the outer parts of the Sun. Such currents could provide the motive power for the strange phenomena which the Sun displays at its surface. Indeed, these phenomena seem to require a free energy source, not just in a thin subphotospheric convective skin, but at some depth. Such a source is hard to find in a purely thermal model. It thus seems that the present model may have advantages in problems apparently quite remote from that of the neutrino emission from which it started.

[3] It was already mentioned above that for M_c in the range 0.3–0.5 $M_\odot$ the value of n lies in the range 2.6–3.1.

[4] ρ on the exterior side of the core boundary is μ_e/μ times ρ on the interior side.

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