Line versus continuum solar neutrinos

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The rate of neutrino line emission is calculated for nuclei that are usually assumed to produce only continuum neutrino emission. A convenient formula is derived that gives for solar interior conditions the ratio of the rates of neutrino line emission to continuum neutrino emission. The only significant line emission from the solar interior is expected from the *pep* and ^{7}Be-electron capture reactions.

I. INTRODUCTION

Many new experimental techniques have been proposed to detect low-energy solar neutrinos and to provide measurements of the energies of individual neutrinos. Promising experiments are being investigated with detectors that use neutrino absorption, neutrino-electron scattering, and coherent scattering. Cryogenic detectors, and scintillators of various types, are being actively explored. This burst of experimental creativity justifies examining the predictions of the standard solar model and the standard theory of electroweak interactions to see if there are additional, nontraditional, neutrino signals that might be detectable from the Sun. In this paper, I calculate the standard-model predictions for neutrino lines.

The standard discussions of nuclear astrophysics describe two-body fusion reactions, some of which produce a continuum spectrum of neutrinos. The best known and most fundamental of the reactions that produce continuum neutrinos is

$$p+p \to {}^2\text{H}+e^+ +\nu_e \quad (q_{2\text{ body}} \le 0.420 \text{ MeV}) . \tag{1}$$

It has been known for some time, however, that the three-body *pep* reaction

$$p+e^- +p \to {}^2\text{H}+\nu_e \quad (q_{3\text{ body}} = 1.442 \text{ MeV}) \tag{2}$$

produces a neutrino that has a well-defined energy, whose spread in energy is determined only by the small Doppler effect and by the practically negligible energy contributed to the recoil of the deuterium nucleus. The ratio of the rate of the *pep* reaction to the rate of the fundamental *pp* reaction can be expressed in terms of functions that are easy to evaluate.[1] The rate for the line-producing *pep* reaction is, for the standard solar model, about two-tenths of a percent of the rate for the more familiar *pp* reaction, which corresponds to small (few percent) contributions to the calculated rates for the chlorine and gallium solar-neutrino experiments.[2]

The other well-known reaction that produces a neutrino line in the solar interior is

$${}^7\text{Be}+e^- \to {}^7\text{Li}+\nu_e . \tag{3}$$

In the Sun, most of the electrons captured by ^{7}Be are continuum electrons, since the bound electrons that are studied in laboratory decays are nearly all ionized in the solar interior. About 89.7% of reactions [Eq. (3)] produce a neutrino with an energy of 0.862 MeV, corresponding to a transition to the ground state of ^{7}Li; the remaining fraction produces a neutrino with 0.384 MeV. Energy production forbids the reaction in which ^{7}Be β decays into ^{7}Li, a positron, and a neutrino with a continuum of energies. The more energetic neutrinos from Eq. (3) contribute, on the basis of the standard model, a significant fraction of the detectable neutrino flux in the ^{37}Cl and ^{71}Ga solar-neutrino experiments.[2]

The ratios of the rates of neutrino-line-producing reactions (with an electron in the initial state) to the corresponding rates for reactions that produce neutrinos with continuous spectra (with the electron moved to the final state where it becomes a positron) were estimated to be small for ^{8}B, ^{13}N, and ^{15}O continuum β decays when the theoretical predictions for the ^{37}Cl solar-neutrino experiment were first worked out.[1]

Most recently, Cheeseman[3] has suggested that the rate of the three-body heep reaction,

$${}^3\text{He}+e^- +p \to {}^4\text{He}^* +\nu_e \quad (q \sim 0.1 \text{ MeV}) , \tag{4}$$

might be very much faster than the (extremely rare) hep reaction

$${}^3\text{He}+p \to {}^4\text{He}+\nu_e \quad (q_{2\text{ body}} \le 18.773 \text{ MeV}) . \tag{5}$$

The hep (and the heep) reactions were not calculated in the original discussion of neutrino lines[1] since both were considered to be negligibly slow. The reason that Cheeseman suggested that the heep reaction could be very fast is that it can populate the excited state of helium at about 20-MeV excitation energy. The rate of the hep reaction is inhibited greatly because the nuclear matrix element leading to the ground state of ^{4}He (the only energetically allowed transition for this reaction) is strongly inhibited,[4] whereas the transition matrix element to the excited state of ^{4}He is large.

If the heep reaction, Eq. (4), were indeed very much faster than the hep reaction, Eq. (5), then taking account of the heep reaction could "solve the solar-neutrino problem."[3] The *pp* chain would, in this scenario, be terminated by the heep reaction. The heep neutrinos are so low in energy that they would not be detected in any of the existing solar-neutrino experiments, including the ^{37}Cl, ^{71}Ga, and Kamiokande II detectors. This is an idea with important implications and, therefore, must be checked quantitatively.

I have been unable to discover in the literature any significant discussion of the emission of solar-neutrino lines in competition with continuum neutrino emission that is subsequent to the detailed but specific calculation of the *pep* reaction in 1969 (Ref. 5). I have therefore rederived the appropriate formulas and present them here in a general form. Using the analytic results, I have calculated the ratio of the neutrino line emission reaction (electron capture) to the usual neutrino continuum producing reaction (with positron emission) for all of the nuclear reactions that produce a significant number of solar neutrinos.

The basic result of the calculations is that for conditions of the solar interior the line production (electron capture) reactions are negligible for all cases except the *pep* and ^{7}Be reactions. The essential reason is that the ratio of line to continuum emission depends upon the ratio of the volume of phase space for one lepton versus the volume of phase space for two leptons. For neutrino energies smaller or comparable to the rest mass of the electron, which is the case for the *pep* reaction, the one-body phase space is relatively large. For large neutrino energies, which is the case for all of the reactions that produce continuum neutrinos except the *pp* reaction, the two-body phase space becomes very large, strongly favoring the emission of a positron and a neutrino rather than the capture of an electron. The results are summarized in the following two sections and in Table I.

II. GENERAL FORMULAS

The general result for the ratio of the rates of line to continuum neutrino emission[1,5] is

$$\frac{R_{\text{line}}}{R_{\text{continuum}}} = 2^{3/2}\pi^{5/2}\alpha n_e Z_{\text{eff}} q_{\text{line}}^2 (kT)^{-1/2} \times f(Z_{\text{final}}, W_0)_{\text{continuum}}^{-1} I(\beta) \times \left[\frac{\Lambda_{\text{line}}}{\Lambda_{\text{continuum}}}\right]^2 , \tag{6}$$

where Z_{final} is the electrical charge on the final nucleus [e.g., $Z_{\text{final}}=1$ for the *pp* reaction of Eq. (1)], and the nuclear energy release W_0 is related to the energy of the neutrino line q_{line} and the maximum neutrino energy for continuum emission $q_{\text{max continuum}}$, by

$$W_0 = q_{\text{line}} - m_e c^2 = q_{\text{max continuum}} + m_e c^2 . \tag{7}$$

The square of the nuclear matrix elements for the transitions can be different for neutrino line emission and for the more usual continuum emission because of energetic considerations. The two matrix elements are different, for example, if the heep reaction of Eq. (4) proceeds to the first excited state of ^{4}He and the hep reaction of Eq. (5) goes to the ground state of ^{4}He. In writing Eq. (6), I have used units in which $\hbar \equiv m_e \equiv c \equiv 1$. The integral in Eq. (6) is

$$I(\beta) = \int_0^\infty dx \exp\{-x[1-\exp(-2.50\beta x^{-1/2})]\} , \tag{8}$$

where

$$\beta \equiv Z_{\text{eff}} T_6^{-1/2} \sim 0.3 Z_{\text{eff}} \tag{9}$$

and T_6 is the temperature in units of millions of degrees. Z_{eff} is the effective charge on the initial nuclear system [e.g., $Z_{\text{eff}}=2$ for the *pep* reaction of Eq. (2)]. Equations (6)–(9) are the general formulas that describe the ratio of neutrino line emission to neutrino continuum emission.

In numerical form, Eq. (6) can be rewritten

$$\frac{R_{\text{line}}}{R_{\text{continuum}}} \approx 1.9\times 10^{-6} \rho (1+X) T_6^{-1/2} Z_{\text{eff}} \times I(Z_{\text{eff}} T_6^{-1/2}) f_{\text{continuum}}^{-1} \left[\frac{q_{\text{line}}}{1\text{ MeV}}\right]^2 , \tag{10}$$

where ρ is the stellar density in units of g cm^{-3}. For typical solar interior conditions,

$$\rho(1+X) T_6^{-1/2} I(\beta) \approx 4\times 10^1 , \tag{11}$$

where X is the hydrogen mass fraction. The above equations can be combined to give a convenient approximate formula for estimating the ratio of line to continuum emission. I find

$$\frac{R_{\text{line}}}{R_{\text{continuum}}} \approx \frac{8 Z_{\text{eff}} \times 10^{-5}}{f(Z_{\text{final}}, W_0)_{\text{continuum}}} \times \left[\left[\frac{q_{\text{line}}}{1\text{ MeV}}\right]\frac{\Lambda_{\text{line}}}{\Lambda_{\text{continuum}}}\right]^2 . \tag{12}$$

The numerical value calculated from Eq. (12) yields a ratio of the *pep* to *pp* neutrino flux that agrees with the results of standard solar-model calculations to within a few percent; the agreement is fortuitously good.

III. NUMERICAL RESULTS

Table I shows the numerical results that are calculated for typical solar interior conditions using the approximate formula given in Eq. (12). For all except the *pep* reaction, the rate of the line neutrino emission is less than a tenth of a percent of the rate of the corresponding contin-

TABLE I. Ratio of three-body to two-body reaction rates.

Reaction (two body)	$R_{\text{line}}/R_{\text{continuum}}$
$p+p \rightarrow {}^2\text{H} + e^+ + \nu_e$	2.3×10^{-3}
${}^3\text{He}+p \rightarrow {}^4\text{He}+e^+ + \nu_e$	4×10^{-8}[a] $\leq 7\times 10^{-7}$[b]
${}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e$	2×10^{-7}
${}^{13}\text{N} \rightarrow {}^{13}\text{C} + e^+ + \nu_e$	4×10^{-4}
${}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^+ + \nu_e$	1×10^{-4}
${}^{17}\text{F} \rightarrow {}^{17}\text{O} + e^+ + \nu_e$	2×10^{-4}

[a]Transition to ground state of ^{4}He.
[b]Transition to excited state of ^{4}He.

uum emission.

The heep reaction, Eq. (4), discussed by Cheeseman,[4] requires special comment since this reaction can be achieved via a transition to either the ground or the first excited state of ^{4}He. The corresponding continuum emission, Eq. (5), can only occur to the ground state of ^{4}He. The nuclear matrix elements cancel out when the heep reaction leads to the ground state of ^{4}He. In this case, the ratio of rates is 4×10^{-8}. As pointed out by Cheeseman,[4] the nuclear matrix element leading to the excited state of ^{4}He is much larger than the suppressed rate to the ground state. I have estimated an upper limit to the ratio of the matrix elements by examining the values of the thermal neutron capture cross sections for many different reactions.[6] These capture cross sections, when divided by the cube of the energy of the emitted γ ray, are approximately proportional to the square of the relevant weak-interaction matrix element.[4] The maximum enhancement due to nuclear matrix elements is estimated to be about 7×10^5, a value used to derive the upper limit to the ratio given in Table I. The enhancement from nuclear matrix elements is almost compensated by the much smaller available two-body phase space, $q^2_{\text{excited state}} \sim 0.01$ MeV^2 instead of $q^2_{\text{continuum}} \sim 3.6\times10^2$ MeV^2.

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[1]J. N. Bahcall, Phys. Rev. **135**, B137 (1964).

[2]J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989).

[3]P. Cheeseman, report, 1989 (unpublished).

[4]P. E. Tegner and C. Bargholtz, Astrophys. J. **272**, 311 (1983).

[5]J. N. Bahcall and R. M. May, Astrophys. J. **155**, 501 (1969).

[6]S. F. Mughabghab and D. I. Garber, *Neutron Cross Sections, Vol. 1, Resonance Parameters* (BNL Report No. 325) (National Neutron Cross Section Center, Brookhaven National Laboratory, Upton, New York, 1973).