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On solar model solutions to the solar neutrino problem

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Without assuming any solar neutrino spectra but merely assuming pure ν_e emissions from the Sun, neutrinos seen by the Kamiokande experiment should produce at least 2.6 ± 0.45 SNU in the lower threshold Homestake experiment. This rate is compared with the total event rate of 2.55 ± 0.25 SNU observed by the Homestake experiment which solar models tell us should measure not only ^{8}B neutrinos seen by the Kamiokande but also uncertainty-free pep neutrinos (which contribute 0.2 SNU) as well as ^{7}Be neutrinos whose energies are below the Kamiokande threshold. This comparison may imply that ^{7}Be neutrinos are more severely suppressed than the ^{8}B neutrinos with respect to the predictions of standard solar models, which cannot be explained by any known astrophysics solution. (In particular, this argument is independent of uncertainties in solar nuclear reaction rates.) It is also noted that the lower limit that the Kamiokande observations set on the ^{8}B neutrino flux restricts variations of standard solar models to require minimal rates of 3.6 SNU for the Homestake experiment and 114 SNU for GALLEX and SAGE to achieve consistency (and still fit helioseismic data). Therefore, variations of standard solar models as solutions to the solar neutrino problem are inconsistent with the Homestake experiment and only marginally allowed by the gallium experiments. If the gallium experiments eventually confirm a flux significantly below 114 SNU, it would seem to imply new neutrino physics.

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I. INTRODUCTION

The observed deficit of the solar neutrino flux with respect to the prediction of the standard solar models has been one of the most interesting problems in both astrophysics and particle physics [1–4]. The so-called "solar neutrino problem" received renewed interest recently following reports of new data possibly conflicting with standard assumptions on the crucial $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction rate [5,6] which led to renewed attempts to solve the problem by modifying the standard solar models [4,7]. In this paper, we attempt to review the current solar neutrino situation in the light of the most recent nuclear experimental results and solar neutrino experimental results, and to examine the minimal rates that successful solar models can yield for the Homestake and the two gallium experiments. We use the results of the Kamiokande experiment without making any connection to solar models to show the reason for the difficulty between the Kamiokande data and the Homestake data. We also note that even with the assumption of a very low $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ rate, the Homestake experiment is still in conflict with variations of standard solar models. We show that the gallium experiments will also be in conflict if they are proven to have fluxes significantly below 114 solar neutrino units (SNU) (1 SNU=10^{-36} capture/target atom sec).

The observed solar neutrino flux of four currently available solar neutrino experiments, the Homestake ^{37}Cl capture experiment [8], the Kamiokande ν-e scattering experiment [9], the GALLEX ^{71}Ga capture experiment [10] and the SAGE ^{71}Ga capture experiment [11], have been summarized in Table I. All of them are lower than the predictions of the most referenced models: the Bahcall and Pinsonneault standard solar model (BPSSM) [1] or the Turck-Chièze and Lopes standard solar model (TLSSM) [2]. It should be noted that alternative solar model calculations give essentially identical results if the same input parameters (nuclear reaction rates, radiative opacities, the heavy element abundance in the Sun, the age of the Sun, etc.) are used [1].

Among the four experiments, only the Kamiokande experiment has been fully calibrated. The two gallium experiments GALLEX and SAGE, although their initial results were seemingly inconsistent [13,14], are increasingly consistent with each other after more than two years of observations. The Homestake experiment, which observed the most significant solar neutrino deficit with respect to the standard solar models and is therefore crucial to the solar neutrino problem, naturally draws questions since so much is currently relying on the experiment. Although no serious questions about the experiment have been proven, some still worry that its event rate seems

TABLE I. Solar neutrino fluxes measured by experiments vs predictions of standard solar models.

	Experimental results	BPSSM [1]	TLSSM [2]	A low flux model [4]
Homestake Expt.	2.55 ± 0.25 SNU	8.0 SNU	6.4 SNU	4.7 SNU
Kamiokande Expt.[a]	0.50 ± 0.04 ± 0.06	1.00	0.75	0.54
Gallium Expt.	79 ± 10 ± 6 SNU (GALLEX) 74 ± 17 ± 10 SNU (SAGE)	132 SNU	123 SNU	117 SNU

[a]Normalized by the prediction of BPSSM.

to be inconsistent with a constant solar neutrino flux, in particular its event rate after 1985–1986 pump failures (2.8 ± 0.3 SNU [15]) is significantly higher than that before (2.1±0.3 SNU [16]). The latter point was elaborated by some attempts to reconcile the solar neutrino experiments and the standard solar models, arguing that a higher Homestake rate after 1986 might not be far below some modified standard solar models [7,17].

However, it should be pointed out that the deviation of the Homestake data from a constant rate is only marginally significant [18–20] and there is no ground other than different rates that discriminates data before 1984 from those after 1986. Different pump configurations and different pumping times have been introduced yielding no obvious change in the capture rate [21]. Various tests by the Davis group also showed no unexpected systematic uncertainties [21]. Furthermore, the rate after 1986 is still statistically consistent with the rate before 1984 within about 2σ. It is therefore completely unjustified to use only the Homestake data after 1986 in discussing the solar neutrino problem since, if the rate did shift, then there are unexpected systematic uncertainties in the experiment, which put the whole experiment into question until it can be resolved.

Attempts to lower the solar neutrino flux predicted by solar models appropriately concentrate on two approaches: (1) lowering the core temperature T_c of the Sun; (2) using a lower $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ rate, because the uncertainties from these two factors are known to be far greater than the uncertainties from the other aspects of solar models [21]. In the context of the standard solar models (i.e., no rotations, no magnetic fields, no exotic particles, and standard nuclear reaction network, etc. [21]), a lower T_c can be achieved by a lower heavy element abundance of the Sun, Z, and/or lower radiative opacities at the center of the Sun. Most if not all nonstandard solar models, such as invoking assumptions such as a 10^9 Gauss magnetic field in the solar interior, or a black hole at the center of the Sun, or capture of weakly interacted massive particles (WIMP's) by the Sun, or even invoking additional core mixing, also end up reducing the neutrino flux by effectively lowering the core temperature T_c of the Sun [21]. They therefore can be categorically included in the first approach to lower the predicted solar neutrino flux.

It has been shown by Bahcall *et al.* that in standard solar models the predicted fluxes from different neutrino sources depend very differently on the core temperature of the models [12,21]. For example, for pp neutrinos, $^7\mathrm{Be}$ neutrinos, and $^8\mathrm{B}$ neutrinos, which are predicted to constitute most of the neutrinos detected in solar neutrino experiments,

$$\phi(pp) \propto T_c^{-1.2}, \quad \phi(^7\mathrm{Be}) \propto T_c^{8}, \quad \phi(^8\mathrm{B}) \propto T_c^{18}, \qquad (1)$$

where ϕ's are neutrino fluxes. Therefore, a ±1.5% variation in T_c alone, which is readily achievable by adjusting standard solar models within a reasonable range (for example, the ±10% 1σ uncertainty in Z alone changes T_c by ±0.8%, as our calculations show approximately that $Z \propto T_c^{0.08}$), will result in a variation in the $^8\mathrm{B}$ solar neutrino flux by a factor of 2. On the other hand, the resultant $^7\mathrm{Be}$ solar neutrino flux only varies by less than 30%, while the predicted pp neutrino flux is essentially free of uncertainties from a variation in T_c in standard solar models.

The uncertainty in the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ rate brings significantly more uncertainty to the $^8\mathrm{B}$ neutrino flux. This rate linearly affects the $^8\mathrm{B}$ neutrino flux (since the $^8\mathrm{B}$ neutrinos are the decay product of the resultant $^8\mathrm{B}$ excited state). Its uncertainty comes from both theoretical extrapolations and interpretations of experimental data themselves [22]. Theoretical extrapolations for six previous experiments yielded $S_{17}(0)$ for the reaction ranging from 16 to 42 eV b, with a weighted average of 22.4±2.1 eV b [22]. [$S(E) = \sigma(E)E\exp(-2\pi\eta)$, where $\sigma(E)$ is the cross section. $\eta = e^2 Z_1 Z_2/\hbar v$ where Z_1 and Z_2 are the charges of colliding particles and v is their relative velocity. The subscript 17 denotes reaction $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$.] Only two of the six experiments, namely those of Kavanagh (1969) [23] and Filippone *et al.* (1983) [24], went to energies well below the $M1$ resonance at ~ 0.6 MeV. They yield 25.2 ± 2.4 eV b (Kavanagh) and 20.2 ± 2.3 eV b (Filippone), respectively, which disagree with each other at 2σ. Several favor the experiment of Filippone *et al.* since it is the latest among the two and was published in a refereed journal [4,17]. The preliminary result of a new experiment by Motobayashi *et al.*, which avoids the $M1$ resonance by using the reverse reaction of disassociating $^8\mathrm{B}$ by the Coulomb field of heavy nuclei, implies a low $S_{17}(0)$. A new extrapolation of the Motobayashi *et al.* data by Langanke and Shoppa [6], which argues for a large $E2$ contribution to the rate, if proven correct, could imply $S_{17}(0)$ as low as 12 ± 3 eV b. Such a low $S_{17}(0)$ would obviously be in conflict with previous measurements. Gai among Motobayashi *et al.*, on the other hand, claims that their data do not favor a large $E2$ contribution [25]. However, given the current situation, one should keep an open mind and possibly allow S_{17} to vary downward by as much as a factor of 2 from the previous average. While such variations can clearly allow easy fits to the Kamiokande data, we will show that they nonetheless do not resolve the apparent conflict between the Kamiokande data and the Homestake data.

II. IMPLICATIONS OF SOLAR NEUTRINO EXPERIMENTS

Given such uncertainties in the ^{8}B neutrino flux, none of the individual solar neutrino experiments taken by themselves indicates strong evidence for a solar neutrino deficit that cannot be resolved with solar model variations: the Kamiokande experiment observes ^{8}B neutrinos only; hence, its event rate may serve better as a normalization of the absolute ^{8}B neutrino flux than an indicator of a solar neutrino deficit; the Homestake experiment, which should observe mostly ^{8}B neutrinos plus some ^{7}Be neutrinos [and a few $p+e^-+p$ (pep) neutrinos and CNO neutrinos], is also subject to a large uncertainty in its expected flux; the two gallium experiments, although capable of observing the uncertainty-free *pp* neutrinos, see a solar neutrino flux that is larger than the *pp* neutrino flux, hence fail to show a deficit in the model-independent *pp* neutrinos.

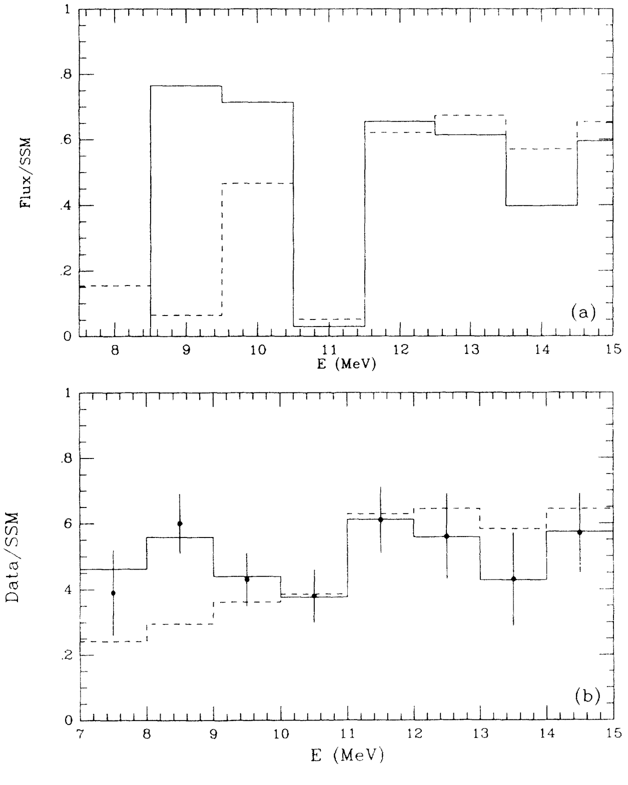

FIG. 1. (a) Neutrino spectrum 1 (the solid line) is the best fit to the Kamiokande data and yields 2.6 SNU to the Homestake experiment (constrained by the condition that the neutrino flux at any energy is non-negative); neutrino spectrum 2 (the dashed line) is excluded by the Kamiokande result at 95% C.L. and yields 1.7 SNU to the Homestake experiment. Both spectra are normalized by the prediction of BPSSM [1]. (b) The solid line is the expected spectrum of recoil electrons from neutrino spectrum 1; the dashed line is the expected spectrum of recoil electrons from neutrino spectrum 2. The Kamiokande data are also shown [41]. Although spectra are normalized by the prediction of the BPSSM for convenience, the results are completely independent of solar models.

Nevertheless, a problem for solar model solutions to the solar neutrino problem may still persist if results from the Kamiokande experiment and the Homestake experiment are combined [4,26–29]. That is, if one normalizes the ^{8}B neutrino flux predicted by solar models to the Kamiokande result, the ^{8}B neutrinos should still contribute 3.1 ± 0.4 SNU to the Homestake result. (An argument that the two experiments with different energy thresholds may observe different reductions in the ^{8}B neutrinos with respect to a standard solar model immediately implies a departure in the ^{8}B neutrino spectrum from that predicted by the standard electroweak theory, and thus new particle physics.) The observed Homestake rate of 2.55 ± 0.25 SNU, therefore, indicates that the ^{7}Be neutrinos which should also be seen by the Homestake experiment, suffer more reduction than the ^{8}B neutrinos with respect to a particular standard solar model, in this case, the BPSSM.

A similar but model-independent argument is also intriguing. With a lower neutrino energy threshold the Homestake experiment should see all the neutrinos observed by the Kamiokande (namely the ^{8}B neutrinos but for this argument the source is irrelevant) and some neutrinos that have energies below the Kamiokande threshold (presumably ^{7}Be neutrinos, pep neutrinos and CNO neutrinos). Without any solar model assumption on the solar neutrino spectrum and flux, but only assuming a pure ν_e flux from the Sun (i.e., no neutrino mixings), solar neutrinos observed by the Kamiokande should contribute 2.6 ± 0.45 SNU to the Homestake result.[1] Figure 1(a) shows a neutrino spectrum that yields the best fit to the Kamiokande data and 2.6 SNU to the Homestake experiment (constrained by the condition that the neutrino flux at any energy is non-negative), and a neutrino spectrum that is excluded by the Kamiokande result at 95% C.L. and yields 1.7 SNU to the Homestake experiment. Figure 1(b) shows their resultant recoil-electron spectra compared with the spectrum observed by the Kamiokande experiment. For convenience, spectra in Fig. 1 are normalized by the prediction of the BPSSM, but the results are completely independent of solar models. The pep neutrinos whose flux is directly tied to the *pp* neutrino flux, which is essentially uncertainty-free, are insensitive to solar model uncertainties and should contribute another 0.2 SNU to the Homestake result [21]. Therefore, the Homestake result of 2.55 ± 0.25 SNU rel-

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[1] While this work was being completed we received a preprint from Kwong and Rosen reporting a similar argument but with a solar model assumption for the neutrino spectrum observed by the Kamiokande [30].

ative to the minimum of 2.6 ± 0.45 SNU from the neutrinos seen by the Kamiokande alone suggests that the ^{7}Be neutrinos may be severely suppressed with respect to predictions of standard solar models. As a result, to reconcile the Homestake result and the Kamiokande result requires either an explanation of a larger reduction in the ^{7}Be neutrinos than that in the ^{8}B neutrinos with respect to predictions of standard solar models, and/or an assumption of contributions from other neutrino flavors to the Kamiokande experiment, either of which would imply new neutrino physics or that one of the two experiments is wrong.

So far, no modification in solar models with known physics can explain a more severe reduction in ^{7}Be neutrinos than in ^{8}B neutrinos with respect to the BPSSM [4,27–29]. Lowering T_c in standard solar models will only suppress more ^{8}B neutrinos since they are more T_c sensitive, as seen from Eq. (1). This conclusion is also valid for nonstandard solar models, since the ^{8}B neutrinos are intrinsically more T_c dependent than the ^{7}Be neutrinos due to a higher Coulomb barrier in the reaction that produces the ^{8}B neutrinos [21]. In terms of nuclear reactions, the $^7\mathrm{Be}(e,\nu_e)^7\mathrm{Li}$ reaction that produces ^{7}Be neutrinos and the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction that produces ^{8}B neutrinos are the two branches of the ^{7}Be reactions in the Sun, with branching ratios of 99.6% and 0.4%, respectively [21]. The only artificial way to achieve a greater reduction in the ^{7}Be neutrinos than in the ^{8}B neutrinos is then to suppress the ^{7}Be production rate (by either suppressing $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$ rate at low energy by at least a factor of 2 or increasing the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ rate at low energy by more than a factor of 4 to account for the ^{8}B neutrinos observed by the Kamiokande [31]) which affects the ^{7}Be neutrinos and ^{8}B neutrino equally, and at the same time increase the $S_{17}(0)$ or raise T_c. But besides the fact that $S_{34}(0)$ and $S_{33}(0)$ (which are astrophysical factors similarly defined as $S_{17}(0)$ but for the $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$ reaction and the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ reaction) are currently determined to within about 10% and 20% respectively [1], the possible trend of $S_{17}(0)$ is downward instead of upward, and a higher T_c will increase CNO neutrinos (which have an even higher T_c dependency than the ^{8}B neutrinos) significantly to escalate the conflict between the Homestake result and the Kamiokande result rather than solve it.

III. VARIATIONS OF STANDARD SOLAR MODELS

To illustrate the above arguments and to show to what extent the solar neutrino prediction of standard solar models can vary, we construct a series of standard solar models with different T_c's and allow $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ to vary freely. These models are constructed with Dearborn's solar code [32] using the Livermore OPAL opacity table and nuclear reaction rates from Caughlan and Fowler (1988) [33] except for a freely varying $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ rate. As a benchmark, we compare one of our models with Bahcall and Pinsonneault's no diffusion model (BPSSM without diffusion) in Table II. They yield quite similar results. To see the uncertainties caused by the $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$ rate and the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ rate, we also calculated solar models with different $S_{34}(0)$ and $S_{33}(0)$, although the variations in solar neutrino fluxes from their uncertainties are relatively small.

TABLE II. Comparison between a model in this work and BPSSM without diffusion [1].

	BPSSM without diffusion	A model in this work
Predictions for ^{37}Cl Expt.		
pep neutrinos	0.2 SNU	0.2 SNU
^{7}Be neutrinos	1.1 SNU	1.1 SNU
^{8}B neutrinos	5.5 SNU	5.5 SNU
^{13}N neutrinos	0.1 SNU	0.1 SNU
^{15}O neutrinos	0.3 SNU	0.4 SNU
Total	7.2 SNU	7.3 SNU
Predictions for ^{71}Ga Expt.		
pp neutrinos	71 SNU	70 SNU
pep neutrinos	3 SNU	3 SNU
^{7}Be neutrinos	34 SNU	33 SNU
^{8}B neutrinos	12 SNU	12.4 SNU
^{13}N neutrinos	3 SNU	3.5 SNU
^{15}O neutrinos	4 SNU	6.6 SNU
Total	127 SNU	129 SNU
Z	0.01895	0.0190
$S_{17}(0)$	22.4 eV b	22.4 eV b
T_c	1.56×10^7 K	1.577×10^7 K

Models with different T_c are constructed by varying Z between 0.014 and 0.021. The measured value of Z is 0.0245 times the solar hydrogen abundance, or roughly 0.0177 ± 0.0017 [34]. Our lowest T_c model has a Z of 0.014, that is more than 2σ below the measured value. Models with such a low Z show distinctive structure differences from models with $Z \sim 0.018$. For example, the model with $Z = 0.014$ has a convective zone with a depth between $0.720R_\odot$ and $0.730R_\odot$ (where $R_\odot$ is the radius of the Sun), much shallower than the measured $0.713 \pm 0.003R_\odot$ from helioseismology [35], whereas models with Z in the range of 0.015 to 0.021 have convective zones as deep as between $0.718R_\odot$ and $0.705R_\odot$. Models with $Z > 0.021$ may also be compatible with helioseismic results but they yield higher neutrino fluxes contrary to the direction of solving the solar neutrino problem.

Figure 2 shows predictions of these solar models with different T_c and $S_{17}(0)$ for the four solar neutrino experiments. Clearly, no overlap region exists between any two experiments at the 2σ level for each experiment, except for a small overlap between the Homestake and SAGE at low T_c and low $S_{17}(0)$. The gap between the Homestake experiment and the Kamiokande experiment is significantly large, not surprisingly as argued before, due to the additional contributions from ^{7}Be neutrinos, CNO neutrinos, and pep neutrinos in the Homestake experiment. In fact, as long as the Homestake experiment result is significantly below 3.6 SNU, a conflict between the Homestake experiment and the Kamiokande experiment cannot be solved by simply lowering T_c and the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ rate in the standard solar models.

The gaps between gallium experiments and the other two experiments are not yet as severe as the gap between the Homestake experiment and the Kamiokande experiment. But they are still problematic for the variations to the standard solar models as we discussed here, if the gallium experiment rate is significantly less than 114 SNU. Current gallium experiment results, therefore, may still be marginally compatible with variations of standard solar models. It will be very interesting to see how the GALLEX rate looks after the effective calibration of GALLEX with $^{51}\mathrm{Cr}$ this year. As statistics improve it may be possible that gallium experiments like the Homestake chlorine experiment will not be compatible with a standard solar model normalized by the Kamiokande result and hence would suggest new neutrino physics.

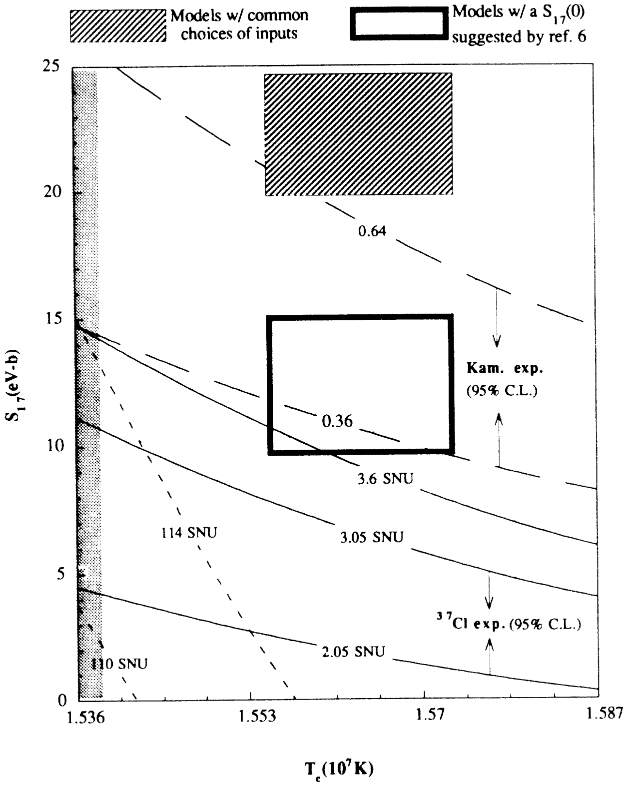

FIG. 2. It shows the predictions of standard solar models with different T_c and $S_{17}(0)$. Long-dashed lines: predictions for the Kamiokande experiment, normalized by the solar model of Bahcall and Pinsonneault [1]; Solid lines: predictions for the Homestake experiment in units of SNU; Short-dashed lines: predictions for gallium experiments in units of SNU. Regions allowed by the Kamiokande experiment and the Homestake experiment at 95% C.L. are shown by arrows. The shaded region on the left side is ruled out by the constraint from helioseismology. The hatched region on the upper center is expected by standard solar models with common choices of inputs (including their uncertainties); the rectangular region at the center is expected by standard solar models with common choices of inputs except a small $S_{17}(0)$ suggested by Ref. [6].

Figure 3 shows predictions of standard solar models with different T_c and artificially varied $S_{34}(0)$ and $S_{33}(0)$. (Solar neutrino fluxes are functions of $S_{34}(0)/\sqrt{S_{33}(0)}$ [21,31].) $S_{17}(0)$ is set to be 20 eV b. Obviously 114 SNU is also the minimal gallium capture rate that can be reached by the standard solar models when allowing S_{34} and S_{33} to vary within their uncertainty ranges and allowing T_c to vary within the constraint from helioseismology. The gap between the Homestake experiment and the Kamiokande experiment cannot be narrowed even with a wild variation of $S_{34}(0)$ by a factor of 2 or $S_{33}(0)$ by a factor of 4.

Besides lowering Z as we did in our solar models, lower T_c may also be equivalently achieved by lowering the overall opacities, increasing the *pp* reaction rate, or shortening the age of the Sun [36]. It is interesting to note that models with a $Z \gtrsim 0.015$ and opacities further artificially lowered only at the center (which might be possible under certain hypotheses [37]) may achieve a T_c lower than the lowest T_c discussed above and still satisfy the current helioseismic constraint. But our calculations show that such models cannot suppress $^7\mathrm{Be}$ neutrinos and CNO neutrinos as efficiently as the simple low Z models when a similar T_c is achieved. In addition, the opacities at

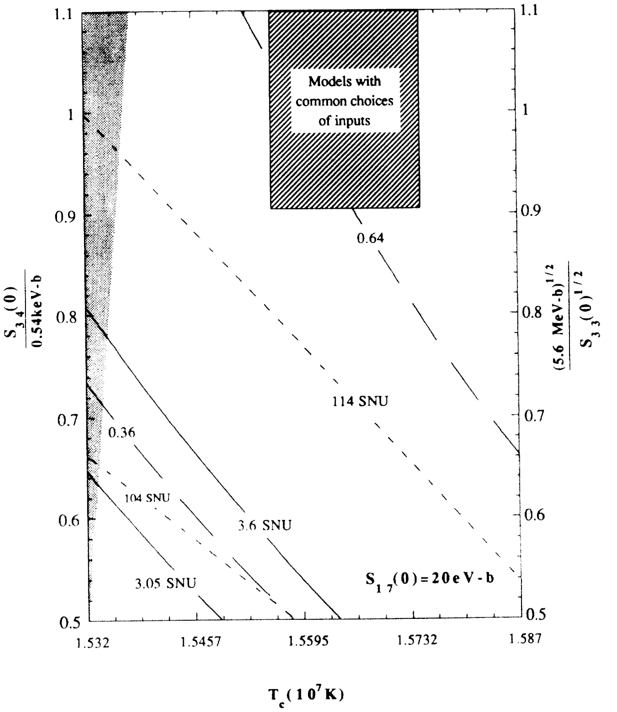

FIG. 3. It shows the predictions of standard solar models with different T_c, $S_{34}(0)$, and $S_{33}(0)$. Long-dashed lines: predictions for the Kamiokande experiment, normalized by the solar model of Bahcall and Pinsonneault [1]; Solid lines: predictions for the Homestake experiment in units of SNU; Short-dashed lines: predictions for gallium experiments in units of SNU. The shaded region on the left side is ruled out by the constraint from helioseismology. The hatched region on the upper center is expected by standard solar models with common choices of inputs (including their uncertainties).

the solar core can only be artificially lowered to the extent that the resultant helium abundance in the Sun is $\gtrsim 0.26$ [38]. As a result, after normalization by the 2σ lower limit of the Kamiokande, the minimal rates predicted by these models for the Homestake experiment and the gallium experiments remain roughly the same as we discussed above.

The result we obtained should also hold for models that include helium diffusion. A comparison between the Bahcall-Pinsonneault helium diffusion model and no helium diffusion model [1] shows that the T_c dependency of various solar neutrino sources [i.e., Eq. (1)] remains roughly intact after consideration of helium diffusion in solar models and the two classes of models yield similar neutrino fluxes when they have the same T_c. Having similar input parameters, a model with helium diffusion yields a higher T_c (and hence higher ^{8}B, ^{7}Be, and CNO neutrino fluxes) than a model without helium diffusion, due to a higher helium concentration in the solar core in the diffusion model that increases the mean molecular weight [1]. Therefore, to achieve the same T_c of a no diffusion model, a helium diffusion model has to have a lower Z input than the no diffusion model. For example, according to our approximate scaling law of $T_c \propto Z^{0.08}$, a helium diffusion model that has the same low T_c as the $Z = 0.014$ no diffusion model should have $Z \approx 0.0135$. On the other hand, since the surface helium abundance of a helium diffusion model is roughly 10% less than the initial helium abundance due to gravitational settling [1], the constraint on Z deduced from the measured Z/X for helium diffusion models becomes stricter than that for no diffusion models, namely $Z \approx 0.0184 \pm 0.0018$ instead of $Z \approx 0.0177 \pm 0.0017$. As a result, helium diffusion models with $Z \approx 0.0135$ are more difficult to reconcile with the constraint on Z. It is also questionable if such a low Z helium diffusion model can satisfy helioseismic constraints.

As we discussed previously, all modifications to solar models that do not contradict known physics can only suppress the ^{7}Be neutrinos by at most the same factor as they suppress the ^{8}B neutrinos with respect to BPSSM. Therefore in the most extreme cases, if the Kamiokande experiment sees only 36% of the BPSSM prediction for ^{8}B neutrinos, the Homestake experiment should then expect 2.7 SNU from the ^{8}B neutrinos and the ^{7}Be neutrinos. If we add the 0.2 SNU contribution from pep neutrinos which has a very small uncertainty [21], and the contribution from CNO neutrinos, which may be neglected since they are very sensitive to T_c and some nuclear rates, we expect an absolute minimal rate of about 3.0 SNU for the Homestake experiment to accommodate any solar model solutions. Similarly, gallium experiments expect 74 SNU from *pp* neutrinos and pep neutrinos, 18 SNU from ^{7}Be and ^{8}B neutrinos, and contributions from CNO neutrinos. Therefore, the minimal gallium rate to accommodate solar models after ^{8}B neutrinos being normalized by the Kamiokande is about 92 SNU if such solar models can successfully suppress most of the uncertain CNO neutrinos. It should be noted, however, we have not been able to construct any realistic solar model that achieves such extreme reductions.

IV. SUMMARY

With an overall average Homestake rate of 2.55 ± 0.25 SNU and a Kamiokande rate of $0.50 \pm 0.04 \pm 0.06$ times the BPSSM prediction, there is little space for a convincing solar model solution to the solar neutrino problem whether or not we assign a solar model spectrum to the neutrinos seen by the Kamiokande experiment, unless one of these experiments is in error. Variations of standard solar models yield a minimal rate of 3.6 SNU for the Homestake experiment and 114 SNU for the gallium experiments, when the ^{8}B neutrino flux is normalized to the Kamiokande result. For gallium experiments, their current yields have not been significantly lower than the minimal 114 SNU. But their accuracy will improve with time and stronger statements may be possible.

It is nice to know the next generation solar neutrino experiments [SNO, Super Kamiokande, Borexino, Imaging of Cosmic and Rare Underground signals (ICARUS), etc. [21]] will be able to distinguish different solutions to the solar neutrino problem within the next five years [39]. And the gallium experiments may even be able to exclude 114 SNU in the not too distant future. Helioseismic observations by the on-going Global Oscillation Network Group (GONG) [40] and future Solar and Heliospheric Observation (SOHO) mission will also provide much more information on the solar interior, thus further constrain solar models.

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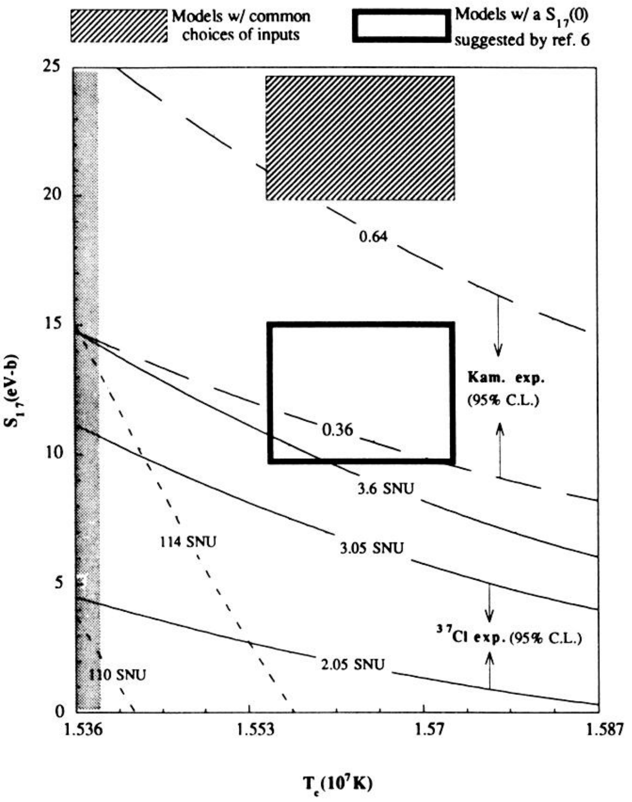

FIG. 2. It shows the predictions of standard solar models with different T_c and $S_{17}(0)$. Long-dashed lines: predictions for the Kamiokande experiment, normalized by the solar model of Bahcall and Pinsonneault [1]; Solid lines: predictions for the Homestake experiment in units of SNU; Short-dashed lines: predictions for gallium experiments in units of SNU. Regions allowed by the Kamiokande experiment and the Homestake experiment at 95% C.L. are shown by arrows. The shaded region on the left side is ruled out by the constraint from helioseismology. The hatched region on the upper center is expected by standard solar models with common choices of inputs (including their uncertainties); the rectangular region at the center is expected by standard solar models with common choices of inputs except a small $S_{17}(0)$ suggested by Ref. [6].

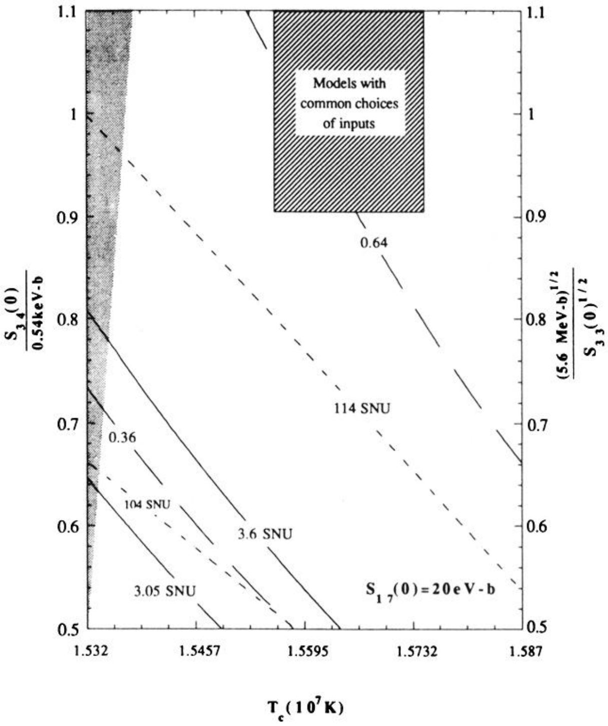

FIG. 3. It shows the predictions of standard solar models with different T_c, $S_{34}(0)$, and $S_{33}(0)$. Long-dashed lines: predictions for the Kamiokande experiment, normalized by the solar model of Bahcall and Pinsonneault [1]; Solid lines: predictions for the Homestake experiment in units of SNU; Short-dashed lines: predictions for gallium experiments in units of SNU. The shaded region on the left side is ruled out by the constraint from helioseismology. The hatched region on the upper center is expected by standard solar models with common choices of inputs (including their uncertainties).