Effect of radiative corrections on the solar neutrino spectrum

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In this paper we calculate the changes to the solar neutrino spectrum arising from the radiative corrections in the β decay processes responsible for the production of the neutrinos. Explicit results are given for the neutrinos arising from the pp reaction and for the ^{8}B neutrinos.

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I. INTRODUCTION

The measurement of solar neutrino fluxes and neutrino energy spectra provides very important information on the processes occurring in the core of the Sun. Present experimental facilities, the gallium detector, the chlorine detector, and the Kamiokande water detector, allow one to measure the total neutrino fluxes arising from specific nuclear processes, such as the pp, the ^{7}Be, and the ^{8}B neutrinos. The measured neutrino fluxes are smaller than what the standard solar model (SSM) leads one to expect. To explain this deficit various proposals have been advanced, some of which involve new physics. Among the nontraditional explanations, the Mikheyev-Smirnov-Wolfenstein matter-induced neutrino oscillations is one of the favorite front runners.

A new generation of solar neutrino detectors is under active consideration now. They will not only measure the total neutrino fluxes but also neutrino energy spectra with an attempt at high statistical accuracy. The neutrino energy spectra are only weakly influenced by the parameters of the SSM and the features of nuclear structure problems. They may be more sensitive to new physics phenomena such as neutrino oscillations. However, if one has experimental results with high statistical accuracy, it will be necessary to take into account a number of corrections to the theoretical neutrino energy spectra before one can check for new physics. Among these new corrections are (i) the distribution of the total β transition energy due to thermal motion of the decaying nuclei which are embedded in the solar plasma, (ii) the contribution of forbidden transitions to the allowed neutrino spectra, and (iii) the contribution of radiative corrections to the β spectra. Of these, the corrections due to (i) and (ii) have been considered by Bahcall and co-workers [1,2]. The last of these, (iii), the effect that the radiative corrections to the β particle has on the neutrino spectrum from β decay, has so far not been calculated. Of course, the radiative corrections to the β particle spectrum in β decays have been reviewed in [3] and have been calculated by a number of people [4–8] and are of the order of 5–6 % near the end-point energy of the β particle for high end-point energies. Based on this one might expect that the neutrino spectrum too would be altered by a correction of a similar magnitude since the β particle and the neutrino are tied together through a sharing of the total energy of the β transition. In this paper we report on the work we have carried out in this regard; actually we find that the corrections to the neutrino spectrum are significantly smaller than those for the electron spectrum. The circumstances which bring this about are commented on in the last section of this paper.

II. RADIATIVE CORRECTIONS AND NEUTRINO ENERGY SPECTRA

Radiative corrections to the electron (positron) spectrum from allowed β decay have been considered in a number of works [4–8]. In obtaining these, integrations have been carried out on the allowed neutrino and photon energies and the results exhibited as a differential spectrum on the β energy. The contributions to the radiative corrections have two components: the emission of real radiation (γ, inner bremsstrahlung) and virtual radiative corrections (v) due to self-energy insertions on the Feynman lines of the charged particles and the vertex corrections. Since the total energy of the β transition is shared by the β particle and the neutrino, any effect on the β particle energy spectrum must have a corresponding effect on the neutrino energy spectrum due to conservation of energy. It would be a simple matter to get the effect on the neutrino spectrum if one has an expression for the triply differential decay rate depending on the energies and momenta of all the concerned particles, namely, the β particle, the neutrino, and the photon.

Following standard methods, details of which we do not present here, we obtain the following expression for the differential rate for the real photon emission process in an allowed β transition:

$$dW^\gamma = \frac{G_F^2}{2\pi^3}\frac{\alpha}{2\pi}(M_F^2 + M_{\rm GT}^2\rho^2)\frac{1}{\epsilon}Q^2dQ\; E_\nu^2 dE_\nu k_1 E_e dE_e dx\; \delta(E_0 - E_e - E_\nu - \epsilon) \times \left[\left(\frac{E_e+\epsilon}{E_e}\right)\frac{\beta^2(1 - x^2Q^2/\epsilon^2)}{(\epsilon - \beta Q x)^2} + \frac{\epsilon}{E_e^2(\epsilon - \beta Q x)}\right]. \tag{1}$$

Here ϵ represents the photon energy, $\epsilon = \sqrt{Q^2+\lambda^2}$, Q being the photon momentum and λ is a small nonzero photon mass for infrared purposes, E_e the electron energy, k_1 the β particle momentum (equal to $\sqrt{E_e^2 - m_e^2}$), $\beta = k_1/E_e$, E_ν the neutrino energy, E_0 the energy of the β transition, M_F and $M_{\rm GT}$ the Fermi and the Gamow-Teller nuclear matrix elements, respectively, and $\rho = g_A/g_V$.

Integrations over the solid angles of the neutrino and of the β particle have been carried out but the angular integration of the photon with respect to the β particle direction ($\cos\theta_{k_1 Q} = x$) has still to be carried out. We have also followed the standard notation G_F for the Fermi coupling constant and α for the fine structure constant.

To obtain the correction to the neutrino energy spectrum, we have to integrate the above expression over the photon energies Q and angles and over the β particle energies E_e. The integration over Q is trivially done with the help of the δ function. The integration over the β particle energies ranges from m_e, the electron mass, to an upper limit $E_0 - E_\nu$. Carrying through the indicated operations we finally get, for the correction due to real emission of radiation,

$$\frac{dW^\gamma}{dE_\nu} = \frac{G_F^2}{2\pi^3}(M_F^2 + M_{\rm GT}^2\rho^2)\frac{\alpha}{\pi}I(E_0 - E_\nu), \tag{2}$$

where I is the integral

$$I(E_0 - E_\nu) = \frac{1}{2}\int_{-1}^{+1} dx \int_{m_e}^{E_0 - E_\nu} dE_e \frac{Q^2}{\epsilon} \times \left[k_1(E_0 - E_\nu)\frac{\beta^2(1 - x^2Q^2/\epsilon^2)}{(\epsilon - \beta Q x)^2} + \beta\frac{\epsilon}{(\epsilon - \beta Q x)}\right]. \tag{3}$$

Here, we still use the notation $Q = E_0 - E_\nu - E_e$ and $\epsilon = \sqrt{Q^2+\lambda^2}$ for compactness of writing. We will continue using these below as an alternative new variable for E_e, the β particle energy.

Now we rewrite the integral over E_e by changing variable to $Q = E_0 - E_\nu - E_e$ as just indicated, and have

$$I(E_0 - E_\nu) = \frac{1}{2}\int_{-1}^{+1} dx \int_0^{Q_{\max}} dQ \frac{Q^2}{\epsilon}\left[(E_0 - E_\nu)\sqrt{(E_0 - E_\nu - Q)^2 - m_e^2}\, F(\beta(Q), x) + \frac{\beta(Q)\epsilon}{\epsilon - \beta(Q)Qx}\right], \tag{4}$$

where $Q_{\max} = E_0 - E_\nu - m_e$, $\beta(Q) = \sqrt{1 - \frac{m_e^2}{(E_0 - E_\nu - Q)^2}}$, and $F(\beta(Q), x)$ stands for the expression

$$F(\beta(Q), x) = \frac{\beta^2(Q)(1 - x^2Q^2/\epsilon^2)}{[\epsilon - \beta(Q)Qx]^2}. \tag{5}$$

As is well known, the integrations over Q and x are very delicate and we need to extract the infrared dependence on λ to cancel with the λ dependence coming from the virtual corrections. In order to perform this, we make use of the evaluation of the double integral over Q and x carried out in Appendix C of the paper by Kinoshita and Sirlin [4]. The first term of our integral, $I(E_0 - E_\nu)$, involving $F(\beta(Q), x)$ is multiplied by the square root factor which depends on Q. If we set $Q = 0$ in this square root factor and replace $F(\beta(Q), x)$ by $F(\beta(0), x)$, we get an integral which is proportional to the integral which Kinoshita and Sirlin have evaluated. This integral in our notation is

$$I_1 = (E_0 - E_\nu)\sqrt{(E_0 - E_\nu)^2 - m_e^2}\,\frac{1}{2}\int_{-1}^{+1} dx \int_0^{Q_{\max}} dQ \frac{\beta^2(0)(1 - x^2Q^2/\epsilon^2)}{[\epsilon - \beta(0)Qx]^2}, \tag{6}$$

and the double integral in this expression is precisely the integral which Kinoshita and Sirlin have given. To make use of this we modify the expression for our integral, $I(E_0 - E_\nu)$, by subtracting and adding the integral I_1 to it as follows. We have

$$I(E_0 - E_\nu) = \frac{1}{2}\int_{-1}^{+1} dx \int_0^{Q_{\max}} dQ \frac{Q^2}{\epsilon}\Bigg[(E_0 - E_\nu)[\sqrt{(E_0 - E_\nu - Q)^2 - m_e^2}F(\beta(Q), x) - \sqrt{(E_0 - E_\nu)^2 - m_e^2}F(\beta(0), x)] + \frac{\beta(Q)\epsilon}{\epsilon - \beta(Q)Qx}\Bigg] + I_1. \tag{7}$$

In this way we have isolated the infrared part in I_1, and the remaining double integral has no divergence in it. This integral can then be done by first performing the integral over x analytically and then doing the Q integral numerically. We find, after integrating over x and calling this function $F(\beta(Q))$ (in the limit $\lambda \to 0$),

$$F(\beta(Q)) = \frac{4}{Q^2}\left[\frac{1}{2\beta(Q)}\ln\left(\frac{1+\beta(Q)}{1-\beta(Q)}\right) - 1\right]. \tag{8}$$

Putting all these together, we get finally the correction to the expression for dW^γ/dE_ν as

$$\frac{dW^{\gamma}}{dE_\nu} = \frac{G_F^2}{2\pi^3}(M_F^2 + M_{\rm GT}^2\rho^2)\frac{\alpha}{\pi}E_\nu^2\Bigg[(E_0 - E_\nu)\sqrt{(E_0-E_\nu)^2 - m_e^2}\left(\frac{1}{\beta(0)}\text{arctanh}\beta(0) - 1\right)$$
$$\times 2\ln\left(\frac{(E_0 - E_\nu - m_e)}{\lambda}\right) + C(\beta(0)) + \frac{1}{2}\int_0^{Q_{\rm max}} QdQ\ln\left(\frac{1+\beta(Q)}{1-\beta(Q)}\right) + \frac{1}{2}\int_0^{Q_{\rm max}} QdQ(E_0 - E_\nu)$$
$$\times\left\{\sqrt{(E_0 - E_\nu - Q)^2 - m_e^2}F(\beta(Q)) - \sqrt{(E_0-E_\nu)^2 - m_e^2}F(\beta(0))\right\}\Bigg], \tag{9}$$

where $C(\beta(0))$ obtained from Kinoshita and Sirlin [4] is

$$C(\beta(0)) = 2\ln 2\left(\frac{1}{\beta(0)}\text{arctanh}\beta(0) - 1\right) + 1 + \frac{1}{2\beta(0)}\text{arctanh}\beta(0)\left[2 + \ln\left(\frac{1-\beta^2(0)}{4}\right)\right]$$
$$+\frac{1}{\beta(0)}[L(\beta(0)) - L(-\beta(0))] + \frac{1}{2\beta(0)}\left[L\left(\frac{1-\beta(0)}{2}\right) - L\left(\frac{1+\beta(0)}{2}\right)\right]. \tag{10}$$

The functions L which appear in this expression are Spence functions with the different arguments indicated.

Now we consider the contribution from the virtual corrections. For this we take the expression from Yokoo *et al.* [7]:

$$\frac{dW^{v}}{dE_\nu} = \frac{G_F^2}{2\pi^3}\frac{\alpha}{\pi}E_\nu^2(E_0 - E_\nu)\sqrt{(E_0-E_\nu)^2 - m_e^2}\left\{(M_F^2 + M_{\rm GT}^2\rho^2)\mathcal{A}(\beta(0)) + M_F^2\left(\frac{3+3\rho}{2}\ln\frac{\Lambda}{M} + \frac{9\rho - 3}{8}\right)\right.$$
$$\left. + M_{\rm GT}^2\rho^2\left(\frac{3+3/\rho}{2}\ln\frac{\Lambda}{M} + \frac{1+5/\rho}{8}\right)\right\}. \tag{11}$$

Here Λ is the renormalization scale (of order nucleon mass), M is the nucleon mass, and $\mathcal{A}(\beta(0))$ stands for the expression,

$$\mathcal{A}(\beta(0)) = \beta(0)[\text{arctanh}\beta(0) - 1] + \left(\frac{1}{\beta(0)}\text{arctanh}\beta(0) - 1\right)2\ln\lambda + \frac{3}{2}\ln\frac{M}{m_e} - \left(\frac{1}{\beta(0)}\text{arctanh}\beta(0) - 1\right)2\ln m_e$$
$$-\frac{1}{\beta(0)}[\text{arctanh}\beta(0)]^2 + \frac{1}{\beta(0)}L\left(\frac{2\beta(0)}{1+\beta(0)}\right). \tag{12}$$

Combining the virtual and the real corrections above, we get for the corrected neutrino spectrum the infrared finite expression

$$\frac{dW^{\rm tot}}{dE_\nu} = \frac{G_F^2}{2\pi^3}E_\nu^2(E_0 - E_\nu)^2\beta(0)(M_F^2 + M_{\rm GT}^2\rho^2)[1 + g(E_\nu, E_0)], \tag{13}$$

where the function $g(E_\nu, E_0)$ is

$$g(E_\nu, E_0) = +\frac{\alpha}{\pi}\Bigg[\left(\frac{1}{\beta(0)}\text{arctanh}\beta(0) - 1\right)2\ln[(E_0 - E_\nu - m_e)/\lambda]$$
$$+C(\beta(0)) + \mathcal{A}(\beta(0)) + \frac{1}{2(E_0-E_\nu)^2\beta(0)}\int_0^{Q_{\rm max}} QdQ\ln\left(\frac{1+\beta(Q)}{1-\beta(Q)}\right)$$
$$+\int_0^{Q_{\rm max}}\frac{QdQ}{2(E_0-E_\nu)\beta(0)}\left[\sqrt{(E_0-E_\nu-Q)^2 - m_e^2}F(\beta(Q)) - \sqrt{(E_0-E_\nu)^2 - m_e^2}F(\beta(0))\right]\Bigg]$$
$$+\frac{\alpha}{\pi}\left(\frac{M_F^2}{(M_F^2 + M_{\rm GT}^2\rho^2)}\left[\frac{3+3\rho}{2}\ln\frac{\Lambda}{M} + \frac{9\rho-3}{8}\right] + \frac{M_{\rm GT}^2\rho^2}{(M_F^2 + M_{\rm GT}^2\rho^2)}\left[\frac{3+3/\rho}{2}\ln\frac{\Lambda}{M} + \frac{1+5/\rho}{8}\right]\right). \tag{14}$$

TABLE I. Effect of radiative correction on the spectrum of boron neutrinos. Here E_ν is the neutrino energy, E_0 is the energy of the β transition, and $g(E_0, E_\nu)$ is the correction from Eq. (13) of the text.

E_ν (MeV)	$g(E_0,E_\nu)$ (%)	E_ν (MeV)	$g(E_0,E_\nu)$ (%)	E_ν (MeV)	$g(E_0,E_\nu)$ (%)	E_ν (MeV)	$g(E_0,E_\nu)$ (%)
0.69	0.62	4.14	0.72	7.60	0.85	11.00	1.08
1.38	0.64	4.83	0.74	8.30	0.89	11.70	1.15
2.07	0.66	5.50	0.77	9.00	0.93	12.40	1.24
2.76	0.68	6.20	0.79	9.70	0.97	13.10	1.37
3.45	0.70	6.90	0.82	10.40	1.02	13.80	1.58

III. DISCUSSION AND CONCLUSION

The correction to the neutrino spectrum represented by the function, $g(E_0, E_\nu)$, in Eq. (13), is unfortunately not integrable analytically. It can, however, be evaluated by carrying out the Q integration numerically. In Table I, the correction $g(E_0, E_\nu)$ is given for different neutrino energies for the boron neutrinos. In Fig. 1 we show a plot of the correction for the case of *pp* neutrinos and for the boron neutrinos. We see from these that the correction does not exceed 2.0% in magnitude at any energy. This is considerably smaller than the 5–6 % effect on the β energy spectrum near the end-point energy. The reason for this seems to be the following.

In the case of the β particle spectrum, the correction term due to the real emission of radiation is positive over the entire energy range of the β particle, while that due to the virtual effects changes sign, being positive for small energies and negative for energies near the end point. These two parts in the total correction reinforce each other for small to intermediate energies, while for energies near the end point the effect of the virtual corrections makes the total correction negative.

In the case of the neutrino, on the other hand, since the total energy of the β and the neutrino is constant, the contribution from the virtual corrections are such that they are *negative for low neutrino energies and positive for energies near the end point.* The contribution from real emission terms is still positive and there is almost complete cancellation between the real and virtual effects in most of the energy region except near the end point where it is small and positive. Thus, while one might naively expect that the correction to the neutrino spectrum must mirror that of the electron spectrum, the actual result for the relative change between low neutrino energies and end point energy is about 2%.

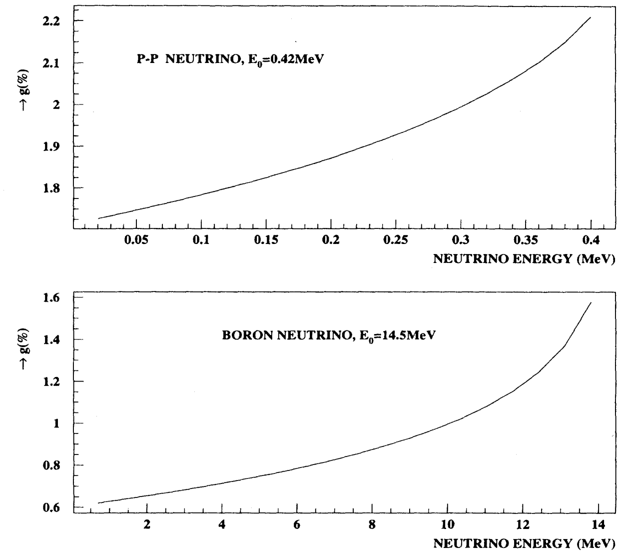

FIG. 1. Radiative correction to neutrino spectrum.

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