Temperature dependence of solar neutrino fluxes

John N. Bahcall
Institute for Advanced Study, Princeton, New Jersey 08540

Andrew Ulmer
Princeton University Observatory, Princeton, New Jersey 08544
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By comparing neutrino fluxes and central temperatures calculated from 1000 detailed numerical solar models, we derive improved scaling laws which show how each of the neutrino fluxes depends upon the central temperature (flux $\propto T^m$); we also estimate uncertainties for the temperature exponents. With the aid of a one-zone model of the Sun, we derive expressions for the temperature exponents of the neutrino fluxes. For the most important neutrino fluxes, the exponents calculated with the one-zone model agree to within 20% or better with the exponents extracted from the detailed numerical models. The one-zone model provides a physical understanding of the temperature dependence of the neutrino fluxes. For the *pp* neutrino flux, the one-zone model explains the (initially surprising) dependence of the flux upon a negative power of the temperature and suggests a new functional dependence. This new function makes explicit the strong anticorrelation between the ^{7}Be and *pp* neutrino fluxes. The one-zone model also predicts successfully the average linear relations between neutrino fluxes, but cannot predict the appreciable scatter in a $\Delta\phi_i/\phi_i$ versus $\Delta\phi_j/\phi_j$ diagram.

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I. INTRODUCTION

Deciphering the solar neutrino problem offers the combined challenge of understanding the structure of the solar interior and understanding the nature of neutrino interactions. The consensus view at present, in part based upon temperature scalings discussed in this paper, is that the measured solar neutrino fluxes reported in the four operating experiments cannot be explained by hypothesizing changes in standard solar models (SSM's). The most plausible explanations, with the currently available data, require some extension of the standard model of electroweak interactions [1–16].

The long-standing discrepancy between the observed and the predicted neutrino fluxes has motivated the study of many nonstandard solar models, which are in most cases *ad hoc* perturbations of the standard solar model. For many of the proposed changes of SSM input parameters (e.g., nuclear cross sections, element abundances, and opacities), the predicted neutrino fluxes are approximately characterized by a single derived model parameter, the central temperature, T. For small variations of input parameters, the neutrino fluxes and the central temperature of a detailed solar model can be related by a power law of the form [17]

$$\phi \propto T^m, \quad m = \frac{d\ \ln\phi}{d\ \ln T}. \tag{1}$$

For the fundamental *pp* neutrinos, $m \sim -1$ and for the important ^{8}B neutrinos, $m \sim +20$.

The temperature dependences indicated in Eq. (1) are obtained from precise calculations with complex stellar evolution codes that solve coupled partial differential equations. The results of these calculations are well known and have been used in many previous analyses of the implications of solar neutrino experiments. Our goal is to show how these simple dependences result from the basic physics of the problem and to what extent the parametrization in terms of a central temperature is sufficient to characterize the neutrino fluxes. The most initially suprising result of the stellar evolution calculations is that the magnitude of the *pp* neutrino flux dependence depends inversely upon the value of the central temperature. We shall see that even this result has a simple, quantitative explanation in terms of the nuclear physics of the energy generation process.

The scaling with the central solar temperature can be used to evaluate neutrino fluxes for small deviations from the standard solar model. If a model is known to have a slightly different central temperature than the SSM, the neutrino fluxes can be estimated without detailed numerical calculations. Many classes of nonstandard models (involving, e.g., rapid rotation in the solar interior, some variations in nuclear cross sections, or the existence of a strong magnetic field in the solar core), reduce the central temperature. A quantitative determination of the reduction in the central temperature implied by a specified change in the input physics requires detailed numerical modeling. With the aid of the temperature scaling laws, neutrino fluxes from nonstandard solar models can be investigated over large parameter ranges.

Using previously determined scaling laws of neutrino fluxes with central temperature, T, [7,17,18] several authors [5,7–10] have studied nonstandard solar models and have compared the predicted neutrino fluxes with the available experimental data from the four operating solar neutrino experiments, Homestake, Kamiokande, GALLEX, and SAGE [19–22]. These authors [5,7–10] show that it is impossible to reconcile the data from the four operating solar neutrino experiments with the neutrino fluxes predicted by changes in T. They conclude that nonstandard solar models which have

TABLE I. The principal reactions of the pp chain.

Reaction	Reaction number	Neutrino energy (MeV)
$p+p\rightarrow{}^2\mathrm{H}+e^+ +\nu_e$	1	0.0 to 0.4
$p+e^- +p\rightarrow{}^2\mathrm{H}+\nu_e$	2	1.4
${}^2\mathrm{H}+p\rightarrow{}^3\mathrm{He}+\gamma$	3	
${}^3\mathrm{He}+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+2p$	4	
or		
${}^3\mathrm{He}+{}^4\mathrm{He}\rightarrow{}^7\mathrm{Be}+\gamma$	5	
then		
$e^- +{}^7\mathrm{Be}\rightarrow{}^7\mathrm{Li}+\nu_e$	6	0.86, 0.38
${}^7\mathrm{Li}+p\rightarrow{}^4\mathrm{He}+{}^4\mathrm{He}$	7	
or		
$p+{}^7\mathrm{Be}\rightarrow{}^8\mathrm{B}+\gamma$	8	
${}^8\mathrm{B}\rightarrow{}^8\mathrm{Be}+e^+ +\nu_e$	9	0 to 14

different central temperatures than the standard model are unlikely to solve the solar neutrino problem.

In the applications described above, the conclusions depend to some extent upon the extrapolation of the temperature scalings to a larger temperature range than was covered in the original numerical models from which the scaling laws were derived [17]. We note that Castellani, Degl'Innocenti, and Fiorentini [7] (see also references [9,10]) find good agreement between the neutrino fluxes calculated from their nonstandard, but detailed, solar models and the fluxes obtained by scaling with respect to the central solar temperature.

The analysis in the present paper provides a physical justification for the use of the temperature scalings, even for relatively large changes in the central solar temperature. We present improved determinations for the temperature exponents and estimates for their uncertainties. We also give exponents for four minor neutrino fluxes for which temperature dependences were not previously available. By construction, the derived scaling law for the fundamental pp neutrino flux is consistent with the observed solar luminosity.

Our results are complementary to the powerful numerical techniques of Castellani *et al.* [7,9,10], who have shown that several nonstandard solar models have a homologous temperature dependence, and to the insightful physical analysis of Bludman [23], who has argued that it is a reasonable approximation to regard the solar interior as a single region that can be largely described by a single parameter, the central solar temperature. Taken together, these previous investigations provide strong motivation for taking seriously the predictions of a single-zone solar model.

In what follows, we shall refer frequently to different fusion reactions in the pp chain. For convenience, we summarize in Table I the principal reactions in the pp chain.

In Sec. II of this paper, we report scaling laws (with uncertainty estimates for the exponents) for all the solar neutrino fluxes that were included in the 1000 numerical solar models calculated in the Monte Carlo study of Bahcall and Ulrich [17]. We present, in Sec. III, a simple one-zone model for the sun which accounts well for the numerically derived scaling laws. This model motivates the use of the new functional form for the temperature dependence of the pp neutrino flux. We discuss in Sec. IV the Fogli-Lisi sum rule [14] on the temperature exponents. In Sec. V, we display the correlations found between the values of the principal neutrino fluxes in the 1000 solar models and show to what extent the one-zone model can account for these correlations. Finally, in Sec. VI we summarize and discuss our main conclusions.

II. TEMPERATURE EXPONENTS: DETAILED SOLAR MODELS

We have derived power-law scaling relations with the central solar (model) temperature, T, for the neutrino fluxes from the 1000 numerical solar models of [17]. The numerical models are calculated with a stellar evolution code (the same code each time) that typically uses a few hundred mass zones. For these precise solar models, the principal input parameters (nuclear cross sections and element abundances) were sampled within their ranges of uncertainty. The average dependences upon the central temperature T of the neutrino fluxes calculated for these models are represented with reasonable accuracy [17] by power-law relations.

Some particle physicists have objected to the term ''1000 standard solar models'' to describe this collection of numerical solar models on the grounds that the same underlying theoretical model (stellar evolution theory and the standard electroweak model) is used to calculate all of the solar models. For their comfort, we have avoided in this paper describing the set of models as ''1000 standard solar models'' and instead refer to them as ''1000 detailed solar models'' or ''1000 numerical solar models.''

In Figs. 1 and 2 and in Table II, we show temperature exponents that were derived by minimizing the residuals in power-law fits of the fluxes versus T. The power-law representations generally describe well the dependences of the neutrino fluxes upon the central temperature of the solar model.

The temperature exponents obtained here by minimizing the residuals for the power-law fits are in approximate agreement with the earlier values [17,18] obtained by best visual fits of power-law temperature dependences to the distribution of neutrino fluxes. The new (previous) temperature exponents of the neutrino fluxes are $m(pp)=-1.1(-1.2)$; $m({}^7\mathrm{Be})=10$ (8); and $m({}^8\mathrm{B})=24$ (18).

For the first time, scaling exponents derived from the Monte Carlo experiment are given here for the less-numerous neutrinos from pep, ${}^{13}\mathrm{N}$, ${}^{15}\mathrm{O}$, and ${}^{17}\mathrm{F}$. In previous applications of the scaling laws, it was necessary to guess the temperature exponents of these four minor neutrino fluxes based upon analogies with the published exponents for the three dominant neutrino fluxes.

We estimate uncertainties in the values of the scaling exponents by determining the indices twice, once by minimizing the residuals in the best fit to the neutrino fluxes and once by minimizing the residuals in the best fit to the central solar temperature. This method attributes all of the scatter in the fits to either the flux or to the central temperature; the calculated range provides a reasonable estimate of the plausible range of the scaling exponents.

Table II presents the best estimates for the scaling expo-

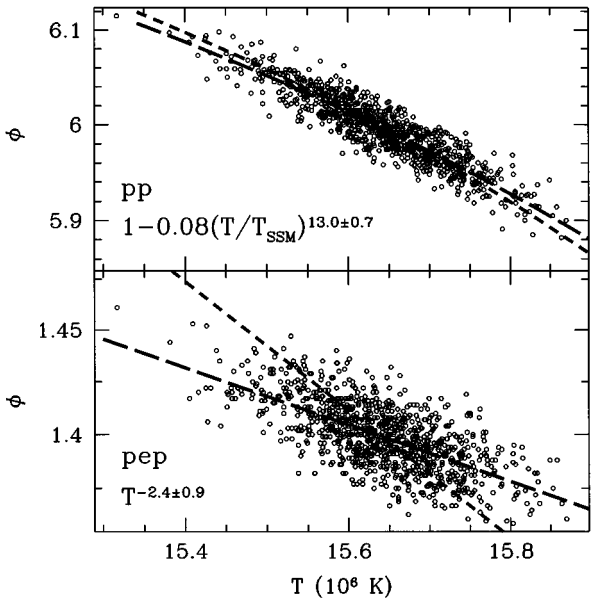

FIG. 1. The pp and pep neutrino fluxes as a function of central solar temperature. The top panel shows the pp neutrino flux versus the central solar temperature (expressed in units of 10^6 K). The lower panel shows the pep neutrino fluxes vs central temperature. The circles correspond to 1000 representations of the standard solar model calculated in a precise Monte Carlo study of the uncertainties in the standard model solar neutrino fluxes [17]. The two plotted lines represent a range of acceptable fits to the numerical data, which correspond to the indicated power-law dependences upon central temperature. The functional form for the pp reaction, $1-0.08(T/T_{\rm SSM}^{n'})$ is discussed in the text.

nents and their estimated errors.[1] We also show in Table II the semianalytic results obtained in the following section using the one-zone solar model. We choose to represent the pp flux with a new functional form, $\phi_{pp} \propto [1-0.08(T/T_{\rm SSM})^m]$, rather than (as has become conventional) $\phi_{pp} \propto T^m$, for the reasons described in the next section.

III. TEMPERATURE EXPONENTS: ONE-ZONE MODEL

Using a static one-zone model of the present-day Sun, we derive in this section approximate scaling laws that agree satisfactorily with the results obtained by detailed evolutionary calculations of numerical solutions of multi-zone solar models. We assume a fixed temperature and matter density for the one-zone solar model.

[1]The residuals were minimized, as presented in Figs. 1 and 2, in logarithmic space. For example, the minimization in temperature was performed on the expression $\Sigma_i|\ln\phi_i - m\ \ln T - \text{const}|$, where ϕ_i is the set of 1000 solar model fluxes. Absolute values were considered to reduce slightly the weight of outliers. The inferred temperature exponents are not sensitive to the precise way the minimization is achieved. The best-estimate exponents given in Table II are the average of the exponents computed by minimizing the residuals in either flux or temperature. The uncertainties presented span the range of the two solutions.

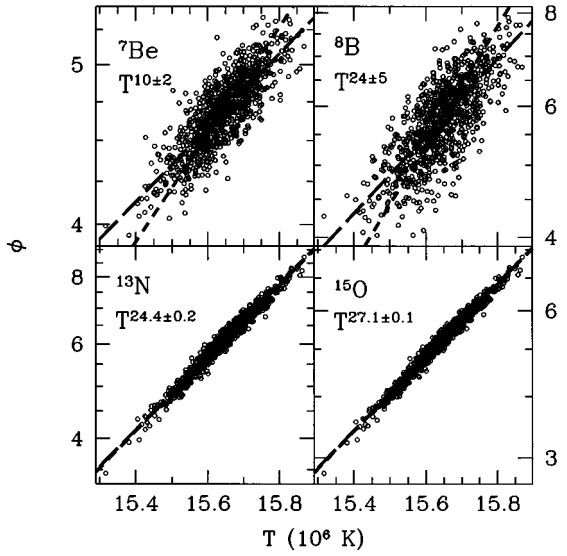

FIG. 2. The ^{7}Be, ^{8}B, ^{13}N, and ^{15}O neutrino fluxes as a function of central solar temperature. The four panels show how the four different neutrino fluxes depend upon central temperature. The circles correspond to 1000 numerical solar models that were computed with different input data [17]. The plotted lines represent a range of acceptable fits to the numerical data, which correspond to the indicated power-law dependences upon temperature.

With this extremely simplified model, we do not expect to obtain accurate values for the temperature scaling laws. However, we shall see that the temperature exponents that are obtained agree surprisingly well with the scaling laws derived from the precise evolutionary solar models.

The last column of Table II gives the temperature exponents obtained in this section for a characteristic one-zone central temperature of $T_c = 14\times10^6$ K. The derived exponents are not particularly sensitive to the assumed characteristic central temperature, T_c. Figure 3 shows the dependence of the exponents obtained with the one zone model for T_c in the range 12×10^6 K to 16×10^6 K.

A. pp and pep temperature exponents

In the one-zone approximation, the measured solar luminosity can be written

TABLE II. Temperature exponents for solar neutrino fluxes. We recommend a new functional dependence for $\phi(pp)$, $\phi(pp) \propto 1-0.08(T/T_{\rm SSM})^{m'}$ as discussed in the text. All other exponents are given for the functional form, $\phi \propto T^m$. For the one-zone model, we assumed a characteristic temperature $T_c = 14\times10^6$ K.

Neutrino flux	Monte Carlo exponent	Estimated uncertainty	One-zone exponent
$\phi(pp),m'$	13.0	0.7	11
$\phi(pp),m$	−1.1	0.1	−0.9
ϕ(pep)	−2.4	0.9	−1.4
$\phi(^7\text{Be})$	10	2	11
$\phi(^8\text{B})$	24	5	25
$\phi(^{13}\text{N})$	24.4	0.2	20
$\phi(^{15}\text{O})$	27.1	0.1	20
$\phi(^{17}\text{F})$	27.8	0.1	23

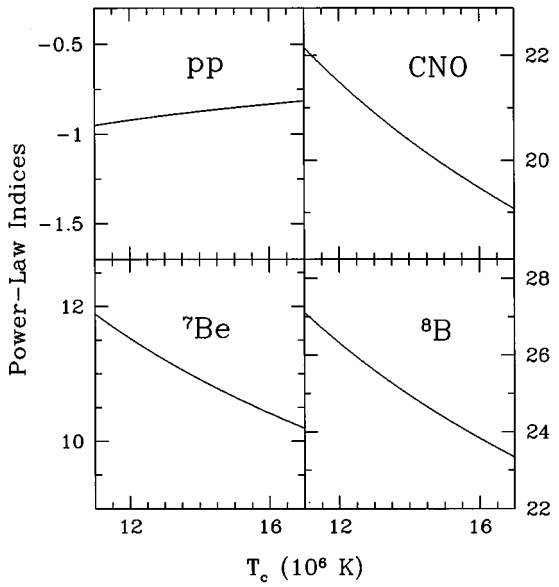

FIG. 3. The power-law exponents. The figure shows how the calculated power-law exponents (m, where neutrino flux $\propto T_c^m$) depend upon the characteristic central temperature of the one-zone model. For example, $m(pp)$ varies between -0.9 and -0.8 as the central temperature is varied between 12×10^6 K and 16×10^6 K.

$$L_\odot = V(\epsilon_{33}R_{33}+\epsilon_{34}R_{34}) \approx V\epsilon_{33}(R_{33}+R_{34}), \tag{2}$$

where V is volume, R is the rate of a nuclear fusion reaction per unit volume, R_{33} corresponds to the nuclear reaction $^3\mathrm{He}+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+2p$ (reaction 4 of Table I), R_{34} corresponds to the reaction $^3\mathrm{He}+{}^4\mathrm{He}\rightarrow{}^7\mathrm{Be}+\gamma$ (reaction 5 of Table I), and ϵ is the amount of energy released by the fusion cycles corrected for neutrino losses. The rates of both R_{33} and R_{34} increase rapidly with temperature; R_{34} increases somewhat faster [18]. For the illustrative purposes of the one-zone model, the 3% difference between ϵ_{33} and ϵ_{34} has been neglected in writing Eq. (2). We have also neglected the small contributions (less than a few percent in standard solar models) to the total luminosity of the fusion reactions associated with the CNO and ^{8}B neutrinos.

The flux of pp neutrinos at Earth is

$$\phi(pp) = \frac{V}{4\pi r_\oplus^2}(2R_{33}+R_{34}). \tag{3}$$

The factor of 2 appears in Eq. (3) because two pp neutrinos are produced in the first branch of the chain; a pp reaction is required to make each of the two ^{3}He nuclei. In the second branch of the pp chain, the α particle acts as a catalyst and therefore only one pp reaction is needed.

Substituting Eq. (2) into Eq. (3), one has

$$\phi(pp) \approx \frac{1}{4\pi r_\oplus^2}\left[\frac{2L_\odot}{\epsilon_{33}} - VR_{34}\right]. \tag{4}$$

Since R_{34} increases with temperature, the temperature dependence of the pp neutrino flux is negative, as was found first in the detailed evolutionary solar model calculations [17].

The appropriate functional relation is, therefore,

$$\phi(pp) = b - aT_c^{m'}. \tag{5}$$

Here $b = 6.5\times10^{10}$ cm^{-2} s^{-1} and

$$m'(pp) = \frac{d\ \ln[n(^3\mathrm{He})n(^4\mathrm{He})\langle 3,4\rangle]}{d\ \ln[T]}. \tag{6}$$

We make use of the convenient notation in which the reaction cross section times velocity averaged over a Maxwell-Boltzman distribution is represented, for the interacting nuclei i and j, by pointed brackets, $\langle i,j\rangle$ (see Eq. 3.9 of [18]). Thus the rate of the $^3\mathrm{He}+{}^4\mathrm{He}$ reaction is represented by $R_{34}=n(^3\mathrm{He})n(^4\mathrm{He})\langle 3,4\rangle$.

Both, the equilibrium number density of $n(^3\mathrm{He})$ and $\langle 3,4\rangle$ are strong functions of temperature. The density can be found [18,24] by solving the equilibrium rate equations. At temperatures representative of the interior of the Sun, a good approximation for the number density of ^{3}He is

$$n(^3\mathrm{He}) \approx n(\mathrm{H})\sqrt{\frac{\langle\ 1,1\ \rangle}{2\langle\ 3,3\ \rangle}}. \tag{7}$$

The rate of the pp reaction is written $R_{1,1}\equiv n(^1\mathrm{H})^2\langle 1,1\rangle/2$, where $\langle i,j\rangle \propto T^{-2/3}\exp(-\tau_{i,j})$, $\tau_{i,j}=3E_{0,ij}/kT$, and E_0 is the most probable energy of interaction [18]. We note that τ is proportional to $T^{-1/3}$.

The value of m' can be calculated from Eq. (6).

Using this one-zone model, we can estimate the value of m' as well as the value of m, which is traditionally used to describe the temperature dependence [see Eqs. (1) and (5)]:

$$m'(pp) \approx 11,$$

$$m(pp) \approx -m'(pp)(aT^{m'}/b) \approx -0.08m'(pp) \approx -0.9, \tag{8}$$

where we have evaluated the exponents at the one-zone characteristic central temperature, $T_c = 14\times10^6$ K. We have also taken $aT^{m'}/b \approx R_{34}/2R_{33}$ to be equal to 0.08, as predicted by detailed numerical solar models or, less precisely, by the one-zone model. The preferred form for the temperature dependence of the pp neutrino flux is, therefore,

$$\phi(pp) \propto [1-0.08(T_c/T_{c,\mathrm{SSM}})^{m'}], \tag{9}$$

where $T_{c,\mathrm{SSM}} = 15.64\times10^6$ K.

The scaling exponent for the pp neutrino flux that is derived from the one-zone model agrees reasonably well with the exponent obtained from precise solar models (see Table II). The Monte Carlo study of 1000 detailed solar models yields $m'=13.0$ ($m=-1.1$). The one-zone model yields $m'=11$ ($m=-0.9$). The one-zone model underestimates the exponent of the temperature dependence of the pp neutrino flux by about 20%.

The exponent for the pep neutrino flux can be obtained with the aid of the analysis of the pp neutrino flux given above, since the rate of the pep reaction (reaction 2 of Table I) depends upon the rate of the pp reaction as [25]: $R(\mathrm{pep}) \propto T^{-1/2}R(pp)$. Therefore, we can write

$$\phi(\mathrm{pep}) \propto T_c^{-1/2}(T_c^m) = T_c^{-1.4}. \tag{10}$$

In the one-zone model, the pep neutrino flux, like the pp neutrino flux, scales like a negative power of the central solar temperature, as is found in the detailed solar model results. The numerical scaling derived from the detailed solar models has a rather large uncertainty, $m(\mathrm{pep}) = -2.4 \pm 0.9$, cf. Table II. Moreover, the power-law exponent is larger for the pep neutrino flux than it is for the pp neutrino flux, which follows from the fact that the pep rate divided by the pp rate is proportional to the modulus squared of the electron wave function near the two protons. The probability density of the electron is inversely proportional to the electron velocity [25], which is itself approximately proportional to $T^{1/2}$.

Although the pep flux could be written in a form similar to Eq. (5) for the pp neutrinos, we have chosen for simplicity to represent this minor component of the solar neutrino flux as a single power of T_c.

Figure 3 shows that the derived scaling exponents for the pp and pep neutrino fluxes do not depend strongly on the value of the temperature, T_c, which is assumed to characterize the solar interior in the one-zone model.

We conclude this subsection by summarizing the main physical insight. The often-quoted dependence of the pp neutrino flux on a negative power of the temperature [$\phi(pp) \propto T^{-1}$] results from the fact that as the central temperature gets larger, an increasing number of the completions of the pp chain proceed through the $^3\mathrm{He}+{}^4\mathrm{He}$ reaction (reaction 5 of Table I). A complete fusion reaction of four protons being converted to an α particle via the $^3\mathrm{He}+{}^4\mathrm{He}$ reaction involves the production of only one pp neutrino (cf. Table I). On the other hand, a fusion of four protons via the $^3\mathrm{He}+{}^3\mathrm{He}$ reaction (reaction 4 of Table I) produces two pp neutrinos. The $^3\mathrm{He}+{}^3\mathrm{He}$ reaction predominates at lower temperatures. Thus the pp flux is larger at lower central temperatures.

B. ^{7}Be and ^{8}B temperature exponents

The dependences of the other neutrino fluxes upon the central solar temperature can all be derived as simple power laws, $\phi \propto T_c^m$. The power-law exponent, m, can be obtained from the one-zone model.

The flux of ^{7}Be neutrinos can be calculated from the rate equation

$$R(^7\mathrm{Be}+e^-) \propto n(e) n(^7\mathrm{Be}) \langle e^-, {}^7\mathrm{Be} \rangle, \tag{11}$$

where $R(^7\mathrm{Be}+e^-)$ is the rate at which ^{7}Be captures electrons in the solar interior (i.e., reaction 6 of Table I). Here $\langle e^-, {}^7\mathrm{Be} \rangle$ is, as before, the reaction cross section times velocity averaged over a Maxwell-Boltzman temperature distribution. The ^{7}Be electron-capture reaction is much faster [18] under solar interior conditions than the competing proton-capture reaction (reaction 8 of Table I). The electron-capture rate is, in equilibrium, essentially equal to the rate of production of ^{7}Be. Moreover, the production rate of ^{7}Be is the same as the reaction rate R_{34} that determines the temperature exponent m' of the pp reaction [see Eq. (4) and the following discussion]. Therefore, the ^{7}Be electron-capture reaction has the same scaling index, $m(^7\mathrm{Be}) = 11$, that was derived in Eq. (8) for the pp reaction. Thus

$$\phi(^7\mathrm{Be}) \propto T_c^{11}, \tag{12}$$

which is approximately 10% larger than the value of $m(^7\mathrm{Be}) = 10$ that is obtained from the detailed numerical models (see Table II).

The temperature dependence of the ^{8}B neutrino flux can be calculated in a similar manner. The ^{8}B neutrino flux results from a rare branch of the pp chain (reaction 8 of Table I) in which ^{7}Be captures a proton rather than an electron. Therefore the ^{8}B neutrino flux can be written as

$$\phi(^8\mathrm{B}) \propto R(^7\mathrm{Be}+e^-) \frac{\langle p, {}^7\mathrm{Be} \rangle}{\langle e^-, {}^7\mathrm{Be} \rangle}. \tag{13}$$

Substituting $T_c = 14 \times 10^6$ K in Eq. (13), we find

$$\phi(^8\mathrm{B}) \propto T_c^{25}. \tag{14}$$

The temperature dependence is much stronger for ^{8}B neutrinos than for ^{7}Be neutrinos because the electron capture rate depends only weakly on temperature (essentially like $T^{-1/2}$ [26]), and the proton capture rate increases rapidly with temperature (like all strong interaction fusion rates).

The derived scaling exponent for ^{8}B, $m(^8\mathrm{B}) = 25$, is fortuitously close to the value of $m = 24$ that is obtained by fitting to the detailed numerical models (cf. Table II).

Figure 3 shows that the derived scaling exponents for the ^{7}Be and ^{8}B neutrino fluxes vary by less than $\pm 10\%$ as the assumed characteristic central temperature T_c varies between 12×10^6 K to 16×10^6 K.

C. CNO temperature exponents

The temperature dependence of the CNO neutrino fluxes can also be estimated simply. For the major part of the CNO cycle which leads from ^{12}C to ^{15}N, the slowest reaction is $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ [18,24]. Therefore, the neutrino fluxes that are produced in this part of the cycle, from ^{13}N and ^{15}O, will both have approximately the same temperature dependence. (Slight differences will occur due, e.g., to nonequilibrium effects not accounted for in our static one-zone model.) The total number of CNO atoms is approximately constant and mostly in the form of ^{14}N.

The temperature exponents for the ^{13}N and ^{15}O neutrino fluxes can be derived from the rate of the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ reaction by calculating the logarithmic derivative of the reaction rate with respect to temperature. Thus $m = \{\tau[^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}] - 2\}/3$. The temperature dependence of the ^{13}N and ^{15}O neutrino fluxes is therefore:

$$\phi(^{13}\mathrm{N}), \phi(^{15}\mathrm{O}) \propto T_c^{20}. \tag{15}$$

The exponents derived above are in reasonable agreement with the scaling laws obtained from the (non-equilibrium) detailed solar models, which are $m(^{13}\mathrm{N}) = 24$ and $m(^{15}\mathrm{O}) = 27$ (see Table II).

Finally, we calculate the temperature dependence of the rare ^{17}F neutrino flux. The slowest reaction involved in producing the ^{17}F neutrinos is $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$. The temperature dependence of the ^{17}F neutrinos can be calculated by analogy with the calculation for the ^{13}N and ^{15}O neutrinos. In the derivation for the ^{17}F neutrinos, we consider the

$^{16}O(p,\gamma)^{17}F$ reaction instead of the $^{14}N(p,\gamma)^{15}O$ reaction. The scaling law derived in this way is

$$\phi(^{17}F) \propto T_c^{23}. \tag{16}$$

This result is in satisfactory agreement with the exponent obtained from the 1000 solar models, which is $m(^{17}F)=28$ (see Table II).

Figure 3 shows that the derived scaling exponents for the CNO neutrino fluxes vary by less than $\pm 10\%$ as the assumed characteristic central temperature T_c varies between 12×10^6 K to 16×10^6 K.

IV. SUM RULE FOR TEMPERATURE EXPONENTS

The luminosity of the Sun can be expressed in terms of the individual neutrino fluxes ϕ_i as [7,14,27,28]

$$L_\odot \propto \Sigma_i \epsilon_i \phi_i, \tag{17}$$

where, for neutrinos from the pp chain,

$$\epsilon_i = 13.366 \text{ MeV} - \langle q_i \rangle. \tag{18}$$

Here $\langle q_i \rangle$ is the average energy loss to the star from neutrinos of type i; the values of ϵ_i can be obtained from Table 3.2 of Ref. [18]. Equations (17) and (18) assume that all the nuclear fusion reactions are in equilibrium, which is a reasonably accurate approximation for all but the CNO neutrinos and, in the outer region of the solar core, the reactions that produce and destroy ^{3}He. For the ^{13}N and ^{15}O neutrinos, Eq. (18) does not apply and the values must be calculated separately from Table 3.3 of Ref. [18]. For these neutrinos, $\epsilon(^{13}N)=3.457$ MeV and $\epsilon(^{15}O)=21.572$ MeV. [Equation (18) would also apply to CNO neutrinos if they were in complete equilbirium.]

Fogli and Lisi [14] pointed out that Eq. (17), when combined with the fact that the present-day solar luminosity is a known constant, implies a sum rule on the temperature exponents. In our notation, the Fogli-Lisi sum rule is

$$\Sigma_i \epsilon_i m_i \phi_i = 0. \tag{19}$$

The Monte Carlo exponents given in Table II satisfy the Fogli-Lisi sum rule to an accuracy of 5% or better [i.e., $(\Sigma_i \epsilon_i m_i \phi_i)/(\Sigma_i \epsilon_i \phi_i)$ is less than 5%]. For the one-zone model, the sum rule is only satisfied to an accuracy of $\sim 20\%$. (If only pp and ^{7}Be neutrinos are considered, the sum rule is satisfied by construction in the one-zone model to the accuracy of our numerical approximations.) Violations of the sum rule in the one-zone model are caused primarily by the fact that different neutrino fluxes are produced in different temperature regions of the sun. If high precision is required, the solar neutrino fluxes cannot be parametrized by a single temperature.

V. CORRELATIONS BETWEEN NEUTRINO FLUXES

In the previous sections, we have concentrated (as have most other investigations of this subject) on the average dependence of individual neutrino fluxes on the central solar temperature. In this section, we focus on the correlations that occur between the deviations of different neutrino fluxes from their average values. Previously we asked: On average, how strongly does a particular neutrino flux depend upon temperature? In this section, we ask: If one neutrino flux is larger than its average value by a specified amount, is a second neutrino flux larger or smaller than its average and, if so, by how much?

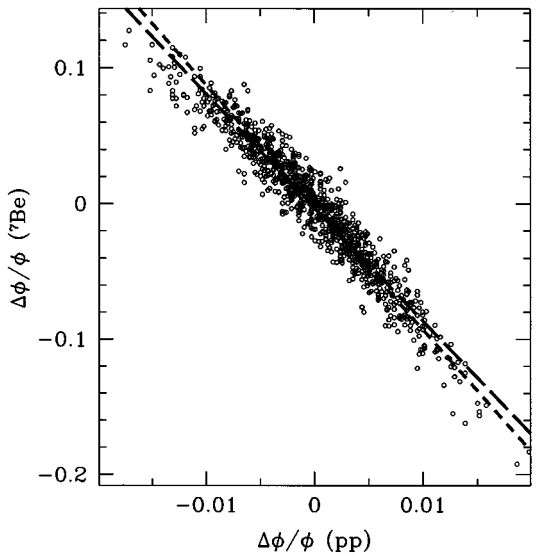

FIG. 4. Correlation of the ^{7}Be and the pp neutrino fluxes. The figure shows the strong correlation, predicted by the one-zone model, between the fractional changes in the ^{7}Be neutrino flux and the fractional changes in the pp neutrino flux as calculated in the 1000 numerical solar models in the Monte Carlo experiment of Ref. [17]. Here $\Delta\phi_i = \phi_i - \phi_{i,\mathrm{SSM}}$.

The one-zone model predicts the relative magnitude and the relative phase of the fractional changes, $\Delta\phi/\phi$, of the ^{7}Be and the pp neutrino fluxes. Here

$$\Delta\phi_i = \phi_i - \phi_{i,\mathrm{SSM}}, \tag{20}$$

where $\phi_{i,\mathrm{SSM}}$ is the standard value of the ith neutrino flux computed for the best input parameters and input physics. Since the temperature dependence of both the ^{7}Be and the pp neutrino fluxes are, in the approximation in which we are working, governed by the rate, R_{34}, of the ^{3}He+^{4}He reaction, the fractional changes in the fluxes are expected to be proportional to each other. The proportionality constant can be derived by comparing Eq. (3), Eq. (4), and Eq. (11). We find

$$\frac{\Delta\phi(^7\mathrm{Be})}{\phi(^7\mathrm{Be})_{\mathrm{SSM}}} = -\left[\frac{\phi(pp)_{\mathrm{SSM}}}{\phi(^7\mathrm{Be})_{\mathrm{SSM}}} + 1\right]\frac{\Delta\phi(pp)}{\phi(pp)_{\mathrm{SSM}}}. \tag{21}$$

Figure 4 shows, as the one-zone model predicts, that the slope of the $\Delta\phi(^7\mathrm{Be})/\phi(^7\mathrm{Be})$ versus $\Delta\phi(pp)/\phi(pp)$ relation is negative and the magnitude of the slope is ~ -10. A closer study of Fig. 4 reveals that the slope, α, obtained with the 1000 numerical models used in the Monte Carlo study is $\alpha_{\text{Monte Carlo}} \approx -9$, whereas the slope predicted by the one-zone solar model is $\alpha_{\text{one zone}} \approx -13$. The difference between the slope obtained with the Monte Carlo study and the slope found with the one-zone model reflects the same imprecision in the one-zone model that was found earlier in Sec. III A and Sec. III B. The slope that is relevant for Fig. 4 is the ratio

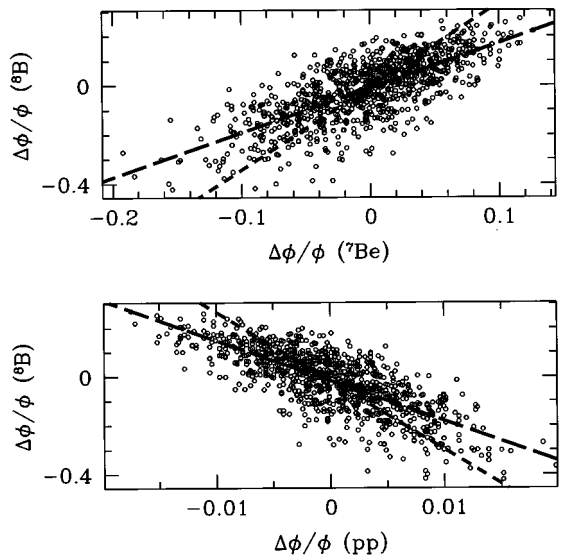

FIG. 5. Correlations between the ^{7}Be, ^{8}B, and pp neutrino fluxes. The figure shows the moderate correlation that exists between the fractional changes in the ^{8}B neutrino flux and the fractional changes of either the ^{7}Be or the pp neutrino flux. Here $\Delta\phi_i = \phi_i - \phi_{i,\mathrm{SSM}}$.

of the ^{7}Be and pp temperature exponents for neutrino fluxes; these exponents are predicted by the one-zone model to be (see Table II), respectively, 10% too large and 20% too small relative to the detailed models.

The one-zone model also predicts the average linear relation between the fractional changes, $\Delta\phi/\phi$, of the ^{8}B, ^{7}Be, and pp neutrino fluxes. Figure 5 shows, in the top panel, the fractional changes in flux for ^{8}B neutrinos versus the fractional changes in flux for the ^{7}Be neutrinos, for the 1000 detailed solar models. The bottom panel of Fig. 5 shows fractional changes in flux for ^{8}B neutrinos versus ^{7}Be neutrinos.

The relevant slope in a plot of $\Delta\phi_i/\phi_i$ versus $\Delta\phi_j/\phi_j$ is just the ratio of the corresponding temperature exponents for ϕ_i and ϕ_j that are given in Table II. Therefore, the predicted one-zone model slope, α, for ^{8}B versus ^{7}Be is $\alpha_{\text{one zone}} \approx -2.3$, which is very close to the value of $\alpha_{\text{Monte Carlo}} \approx -2.4$ found in the Monte Carlo study. The one-zone model predicts a somewhat too steep dependence of fractional changes in ^{8}B neutrino fluxes versus fractional changes in ^{7}Be neutrino fluxes, namely, $\alpha_{\text{one zone}} \approx 28$, versus $\alpha_{\text{Monte Carlo}} \approx 22$.

Figure 5 shows a large scatter in the relation between fractional changes of the ^{8}B neutrino flux and either the ^{7}Be or the pp neutrino flux. This lack of tightness in the relations shown in Fig. 5 results ultimately from the fact that, in all modern solar models, the ^{8}B-producing reaction, reaction 8, Table I, is rare and does not influence significantly the structure of the Sun. In fact, the largest uncertainty in the model calculations of the ^{8}B neutrino flux is caused by the uncertainty in the experimental value for the low-energy nuclear cross section of reaction 8; the value of this cross section has essentially no effect ($<0.1\%$) on the calculated rates of the other nuclear fusion reactions.

VI. CONCLUSION AND DISCUSSION

The most interesting result of our study is the understanding it provides of the negative temperature dependence of the pp neutrino flux. The empirical fact that the pp flux decreases with increasing central temperature, contrary to the trend found with all other solar neutrino fluxes, has been known since 1988 [17,18], but has not previously been explained physically. At first glance, this negative temperature dependence is counterintuitive.

In Sec. III A, we show that the negative temperature dependence is a simple consequence of the fact that at higher temperatures only one pp neutrino is produced per (approximately) 25 MeV communicated to the star (via fusion of four protons), whereas at lower temperatures two pp neutrinos are produced per 25 MeV. In other words, at lower temperatures the ^{3}He-^{3}He fusion termination reaction (which requires two pp reactions) predominates whereas at higher temperatures the ^{3}He-^{4}He reaction is faster (and requires only one pp reaction). The total energy per unit time communicated to the star must equal the observed solar luminosity, independent of the assumed central temperature. Thus, as the temperature increases and more of the nuclear fusion is accomplished by the ^{3}He-^{4}He reaction, fewer pp neutrinos are produced (and more ^{7}Be and ^{8}B neutrinos are created).

In order to obtain a simple physical understanding of the temperature scalings and the correlations between the different neutrino fluxes, we have adopted a one-zone model for the interior of the sun. This model is characterized by a fixed central temperature, T_c, and a total luminosity that is equal to the observed solar luminosity. Given the emphasis in the current literature on calculating ever more precise solar models, with hundreds of different mass shells, it is gratifying and surprising that the one-zone model accounts semiquantitatively for some of the most often used results of the detailed model calculations. Moreover, the one-zone model predicts the average correlations found between the different neutrino fluxes, a bonus in insight that was not possible to anticipate without detailed study of the simple model.

Figure 1 and Fig. 2 display the dependence upon central temperature of the 1000 detailed solar models used in the Bahcall-Ulrich Monte Carlo study [17,18] of theoretical uncertainties in the predicted solar neutrino fluxes. We have used these data to determine average temperature exponents, m, for all of the solar neutrino fluxes, where by assumption $\phi \propto T^m$. The exponents determined here are obtained by a formal best-fitting technique and are to be preferred to the previously estimated exponents [17,18] inferred less formally from these same data; the previously estimated exponents have been widely used in the literature. We have also estimated, for the first time so far as we know, uncertainties in the inferred temperature exponents.

Figure 4 and Fig. 5 show the correlations, found in the Monte Carlo study, between the different individual neutrino fluxes. These correlations reflect the fact that when one neutrino flux is increased or decreased, there is likely to be a corresponding change in the values of the other neutrino fluxes. These correlations must be taken into account when comparing the results of theoretical solar model calculations, including their uncertainties, with solar neutrino experiments. The only precise way to include the correlations dis-

played in Figs. 4 and 5 is to use the complete set of calculated neutrino fluxes in the theoretical analysis (cf. [29]). Various practical approximations to this rather cumbersome method have been discussed in the literature (see, for example, [6,8,14]).

The temperature exponents calculated with the aid of the one-zone model agree with the exponents inferred from the Monte Carlo study of precise solar models to an accuracy of 20% or better for the three most important solar neutrino fluxes: pp, ^{7}Be, and ^{8}B. The results are shown in Table II, which compares the exponents calculated with the one-zone model with the results obtained from the detailed solar models. Figure 3 shows that the scaling exponents calculated in the one-zone solar model are not strongly dependent upon the assumed characteristic central temperature, T_c (taken here to be $T_c = 14 \times 10^6$ K).

The quantitative agreement between the results of the one-zone model and the detailed models is impressive given the fact that the temperature exponents vary from $m \sim -1$ for pp neutrinos to $m \sim +24$ for ^{8}B neutrinos.

The physical insight provided by the one-zone model suggests a new form for the temperature dependence of the pp neutrino flux, which is given in Eq. (9). In this form, the variation of the pp neutrino flux is, for all temperatures, consistent with the observed solar luminosity, since it was derived by considering the relation between the solar luminosity, Eq. (2), and the pp neutrino flux, Eq. (3). Moreover, the formula for the pp neutrino flux, Eq. (4), provided by the one-zone model makes explicit the close correlation between the ^{7}Be and pp neutrino fluxes that is manifest in Fig. 4. The expression used in this paper for the pp neutrino flux, Eq. (9), was derived by considering the relation between the solar luminosity, Eq. (2), and the pp rate, Eq. (3). Physically, the strong correlation exists because the ^{7}Be neutrino flux is proportional to the rate of reaction 5 of Table I and the pp neutrino flux is proportional to a constant minus the rate of reaction 5 (if we neglect the small contribution from CNO neutrinos).

The one-zone model also accounts quantitatively for the average correlation, shown in Fig. 5, between the ^{8}B and ^{7}Be neutrino fluxes, and between the ^{8}B and pp neutrino fluxes.

No simple model can, however, account in detail for the scatter in the correlation plots shown in Figs. 4 and 5. The Monte Carlo experiments simulate uncertainties in many different parameters; the power-law fits in the figures represent only the average response of the neutrino fluxes to the changes in all the individual parameters. For analyses requiring a precise assessment of the correlations between the different neutrino fluxes, a Monte Carlo study of detailed solar models is required.

What have we learned from this study? Improved temperature exponents for the neutrino fluxes are now available, with estimates for the uncertainties in the exponents. A static one-zone model of the sun accounts for the essential features of the temperature scaling of the neutrino fluxes and even describes well the average correlations between the fluxes. The model does not provide a precise description of the temperature dependences nor of the correlations between the different fluxes. The exponents derived from the one-zone mode model do not satify precisely the sum rule derived from the measured solar luminosity.

The fundamental reason that the one-zone model does not account accurately for all of the known results is that in precise solar models each neutrino flux is produced in a different range of temperatures. One cannot represent the results of different temperature ranges by a single parameter, T_c.

In the future, a new Monte Carlo study must be undertaken to determine the temperature scalings and the correlations between the neutrino fluxes when, as required by helioseismological measurements [30], diffusion is taken into account in the solar model calculations. The analysis of Bludman [23] suggests that the effects of diffusion may alter the inferred temperature exponents by a non-negligible amount when compared to the values given in this paper, which are obtained from detailed solar models that do not include diffusion [17]. A Monte Carlo study is now underway that will create 1000 solar models that include diffusion and other recent refinements of the stellar model [31].

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